INTRODUCTION TO MAPLE

Maple is a powerful software system for performing many different symbolic and numerical calculations in mathematics. This document will give a brief introduction to using Maple for calculations in linear algebra. There is a much more complete course on Maple at

`https://neilstrickland.github.io/maths_with_maple/`

Using Maple

When you first start Maple, it will offer you a choice between Document Mode and Worksheet Mode. I recommend Worksheet Mode. When Maple has started you can enter `2+2` and press `ENTER`; Maple will print 4 as an answer. Now enter

```
A := <<5|6|7>,<4|3|2>>
B := <<8|9>,<9|8>>
```

Maple will display this as

$$A := \begin{pmatrix} 5 & 6 & 7 \\ 4 & 3 & 2 \end{pmatrix}$$

$$B := \begin{pmatrix} 9 & 5 \\ 5 & 9 \end{pmatrix}$$

(The colon before the equals sign is essential.) You can now enter `B.A` to calculate BA, or `B^2` to calculate B^2, or `B^(-1)` to calculate the inverse of B. If you enter `A.B` then Maple will print an error message explaining that AB is undefined, because A and B do not have compatible shapes.

The 3×3 identity matrix (for example) can be entered as `IdentityMatrix(3)`. To save typing, it is best to enter

```
I3 := IdentityMatrix(3)
```

and then you can just type `I3` after that. You might think of using the symbol `I` instead of `I3`, but Maple will not allow that, because it uses `I` for $\sqrt{-1}$.

To do more complicated things, you need some additional packages that are not loaded by default. I recommend that as soon as you start Maple you enter

```
with(LinearAlgebra):
with(Student[LinearAlgebra]):
```

This will load up everything that you need. (Here and elsewhere in Maple, you need to use capital letters exactly as specified; it will not work to enter `With(linearalgebra)`.)

In particular, after you have done the above you can enter

```
Transpose(A)
Determinant(B)
```

to calculate A^T and $\det(B)$. You can extract rows and columns from a matrix using syntax like

```
Row(A,2)
Column(B,1)
```

You can refer to the bottom right hand entry of A as `A[2,3]`.

In many places, it is convenient to give names to the columns of a matrix. For example, we might take u_1, u_2 and u_3 to be the columns of the matrix A above. To do this in Maple you can type

```
u[1] := Column(A,1)
u[2] := Column(A,1)
u[3] := Column(A,1)
```

or

```
for i from 1 to 3 do u[i] := Column(A,i) od
```

(Here `od` is `do` backwards, and marks the end of the `do` statement. You can type `end do` instead of `od` if you prefer.)

To find the RREF of A, you can enter

```
ReducedRowEchelonForm(A)
```

That is quite a lot of typing, so I suggest that after loading the `LinearAlgebra` and `Student[LinearAlgebra]` packages you immediately enter

```
RREF := RowReducedEchelonForm
GJET := GaussJordanEliminationTutor
```

You can then type `RREF(A)` to find the RREF of A immediately, or `GJET`to open a new window that leads you through the process step by step.

If you are going to use the RREF of A in further calculations, then you probably want to give it a name, like this:

```
A1 := RREF(A)
```

Again, the colon before the equals sign is necessary.

To find and factor the characteristic polynomial of B you can enter

```
p := CharacteristicPolynomial(B,t)
factor(p)
```

This works as written for square matrices like B that have an even number of rows. However, Maple defines the characteristic polynomial of M to be $\det(M - tI)$, whereas the notes use $\det(tI - M)$. (Both of these conventions are common in the literature.) Thus, for square matrices of odd size you need to multiply Maple's characteristic polynomial by -1 to get the characteristic polynomial as defined in the notes.

To find the eigenvalues and eigenvectors of B, enter `Eigenvectors(B)`. The result consists of a vector and a matrix, like this:

$$\begin{pmatrix} 4 \\ 14 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$$

The eigenvalues are 4 and 14, which appear as the entries in the vector above. The corresponding eigenvectors are $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, which appear as the columns of the matrix above. In cases where the matrix has repeated eigenvalues the result will need a bit more interpretation, but that will be discussed later.