Introduction to Maple

Exercise 2.1

```
> 2+2;
```

$$4 \tag{1}$$

```
> (3+7+10)*(1000-8)/(900+90+2) - 17;
```

$$3 \tag{2}$$

Exercise 2.2

(a)

```
> 6^20 * 15^20 / 9^20;
```

$$100000000000000000000 \tag{3}$$

The answer is one followed by 20 zeros, or in other words 10^{20}. This is easy to see by hand, because $\frac{6^{20}\,15^{20}}{9^{20}} = \left(\frac{6\,x\,15}{9}\right)^{20}$ and (6 x 15)/9 is just 10.

(b)

```
> (10^10 - 1)/99;
```

$$101010101 \tag{4}$$

The answer is $1 + 100 + 10000 + 1000000 + 100000000$, or in other words $1 + 100 + 100^2 + 100^3 + 100^4$. The standard geometric progression formula says that this is the same as $\frac{100^5 - 1}{100 - 1}$, or in other words $\frac{10^{10} - 1}{99}$.

(c)

```
> (10^10 - 10 - 9^2)/9^2;
```

$$123456789 \tag{5}$$

To check this by hand, let x be the number 123456789. Then $10\,x = 1234567890$ and so $10\,x + 9 = 1234567899$. If we subtract x from this the digits mostly cancel and we get $9\,x + 9 = 1111111110$. Multiply by 9 again to get $9^2\,(x + 1) = 9999999990$, which is $10^{10} - 10$. Rearrange this to get $x + 1 = \frac{10^{10} - 10}{9^2}$ and so $x = \frac{10^{10} - 10 - 9^2}{9^2}$.

(d)

```
> (10^9+1)*(10^10 - 10 - 9^2)/9^2;
```

$$123456789123456789 \tag{6}$$

Assuming part (c), we have $\frac{10^{10} - 10 - 9^2}{9^2} = 123456789$ and so $\frac{10^9\,(10^{10} - 10 - 9^2)}{9^2} = 123456789000000000$. These two numbers can be added without any carrying, to give

$$\frac{(10^9+1)\,(10^{10}-10-9^2)}{9^2} = 123456789000000000 + 123456789 = 123456789123456789.$$

Exercise 2.3

The numbers in this exercise are approximations to Pi; in a certain sense, they are actually the best possible approximations.

```
> 3 + 1/7;
```

$$\frac{22}{7} \tag{7}$$

```
> evalf(%);
```

$$3.142857143 \tag{8}$$

```
> 3 + 1/(7 + 1/15);
```

$$\frac{333}{106} \tag{9}$$

```
> evalf(%);
```

$$3.141509434 \tag{10}$$

```
> 3+1/(7+1/(15 + 1/(1 + 1/293)));
```

$$\frac{104348}{33215} \tag{11}$$

```
> evalf(%);
```

$$3.141592654 \tag{12}$$

```
> evalf(Pi);
```

$$3.141592654 \tag{13}$$

Exercise 2.4

```
> evalf[4997](exp(1));
```

```
2.718281828459045235360287471352662497757247093699959574966967627724076630353\5 (14)
    47594571382178525166427427466391932003059921817413596629043572900334295260\
    59563073813232862794349076323382988075319525101901157383418793070215408914\
    99348841675092447614606680822648001684774118537423454424371075390777449920\
    69551702761838606261331384583000752044933826560297606737113200709328709127\
    44374704723069697720931014169283681902551510865746377211125238978442505695\
    36967707854499699679468644549059879316368892300987931277361782154249992295\
    76351482208269895193668033182528869398496465105820939239829488793320362509\
    44311730123819706841614039701983767932068328237646480429531180232878250981\
    94558153017567173613320698112509961818815930416903515988885193458072738667\
    38589422879228499892086805825749279610484198444363463244968487560233624827\
```

0419786232090021609902353043699418491463140934317381436405462531520961836 9\
0888707016768396424378140592714563549061303107208510383750510115747704171 8\
9861068739696552126715468895703503540212340784981933432106817012100562788 0\
2351930332247450158539047304199577770935036604169973297250886876966403555 7\
0716226844716256079882651787134195124665201030592123667719432527867539855 8\
9448969709640975459185695638023637016211204774272283648961342251644507818 2\
4423529486363721417402389344124796357437026375529444833799801612549227850\
9257782562092622648326277933386566481627725164019105900491644998289315056 6\
0472580277863186415519565324425869829469593080191529872117255634754639644 7\
9101459040905862984967912874068705048958586717479854667757573205681288459 2\
0541334053922000113786300945560688166740016984205580403363795376452030402 4\
3225661352783695117788386387443966253224985065499588623428189970773327617 1\
7839280349465014345588970719425863987727547109629537415211151368350627526 0\
2326484728703920764310059584116612054529703023647254929666938115137322753 6\
4509888903136020572481765851180630364428123149655070475102544650117272115 5\
5194866850800368532281831521960037356252794495158284188294787610852639813 9\
5599006737648292244375287184624578036192981971399147564488262603903381441 8\
2326251509748279877799643730899703888677822713836057729788241256119071766 3\
9465070633045279546618550966661856647097113444740160704626215680717481877 8\
4437143698821855967095910259686200235371858874856965220005031173439207321 1\
3908032936344797273559552773490717837934216370120500545132638354400018632 3\
9914907054797780566978533580489669062951194324730995876552368128590413832 4\
1160722602998330535370876138939639177957454016137223618789365260538155841 5\
8718692553860616477983402543512843961294603529133259427949043372990857315 8\
0290958631382683291477116396337092400316894586360606458459251269946557248 3\
9186564209752685082307544254599376917041977780085362730941710163434907696 4\
2372229435236612557250881477922315197477806056967253801718077636034624592 7\
8778465850656050780844211529697521890874019660906651803516501792504619501 3\
6658543663271254963990854914420001457476081930221206602433009641270489439 0\
3971771951806990869986066365832322787093765022601492910115171776359446020 2\
3249300280401867723910288097866605651183260043688508817157238669842242201 0\
2495055188169480322100251542649463981287367765892768816359831247788652014 1\
1741109136011649950766290779436460058519419985601626479076153210387275571 2\
6992518275687989302761761146162549356495903798045838182323368612016243736 5\
6984670378585330527583333793990752166069238053369887956513728559388349989 4\
7074161815501253970646481719467083481972144888987906765037959036696724949 9\

```
2545279033729636162658976039498576741397359441023744329709355477982629614\
9144293645142861715858733974679189757121195618738578364475844842355558105\
0256114923915188930994634284139360803830916628188115037152849670597416256\
8236092168075150177725387402564253470879089137291722828611515915683725241\
3077225440633787593105982676094420326192428531701878177296023541306067213\
0460003896610936470951414171857770141806064436368154644400533160877831431\
4440811949422975599314011888683314832802706553833004693290115744147563139\
9722170380461709289457909627166226074071874997535921275608441473782330327\
3301682371936480021732857349359475643341299430248502357322145978432826414\
1684878721673367010615094243456984401873312810107945127223737886126058165\
6805371439612788873252737389039289050686532413806279602593038772769778379\
8684093253658807339884572187460210053114833513238500478271693762180049047\
5597959290591655470505777514308175112698985188408718564026035305583737832\
2292418562564425502267215598027401261797192804713960068916382866527700975\
7670697770364392602243728418408832518487704726384403795301669054659374616\
9323840363893131364327137688841026811219891275223056256756254701725086349\
6536728860596675274086862740791285657699631378975303466061666980421826772\
5605306607738996242183408598820718646826232150802882863597468396543588566\
5503773131296587975810501214916207656769950659715344763470320853215603674\
2860837865680307306265763346977429563464371670939719306087696349532884683\
61303882943104080029687386911706666
```

Exercise 2.5

```
> Digits := 40;
```

$$Digits := 40 \quad (15)$$

```
> x := exp(Pi*sqrt(163));
```

$$x := e^{\pi\sqrt{163}} \quad (16)$$

```
> y := evalf(x);
```

$$y := 2.625374126407687439999999999992500725944\ 10^{17} \quad (17)$$

```
> printf("%20.20f\n\n",y);
262537412640768743.99999999999925007259
```

```
> z := round(y);
```

$$z := 262537412640768744 \quad (18)$$

```
> y-z;
```
$$-7.499274056\ 10^{-13} \tag{19}$$

```
> restart;
```

Exercise 3.1

```
> A := (x^2-4*y^2)*(x^3-x*y^2);
```
$$A := (x^2 - 4\,y^2)\,(x^3 - x\,y^2) \tag{20}$$

```
> simplify(A);
```
$$x^5 - 5\,x^3\,y^2 + 4\,x\,y^4 \tag{21}$$

```
> expand(A);
```
$$x^5 - 5\,x^3\,y^2 + 4\,x\,y^4 \tag{22}$$

```
> factor(A);
```
$$(x - 2\,y)\,(x + 2\,y)\,x\,(x - y)\,(x + y) \tag{23}$$

```
> convert(expand(A),horner,x);
```
$$(4\,y^4 + (x^2 - 5\,y^2)\,x^2)\,x \tag{24}$$

I would vote for $(x - 2\,y)\,(x + 2\,y)\,x\,(x - y)\,(x + y)$ as the most useful version, but of course it depends what you want to use it for.

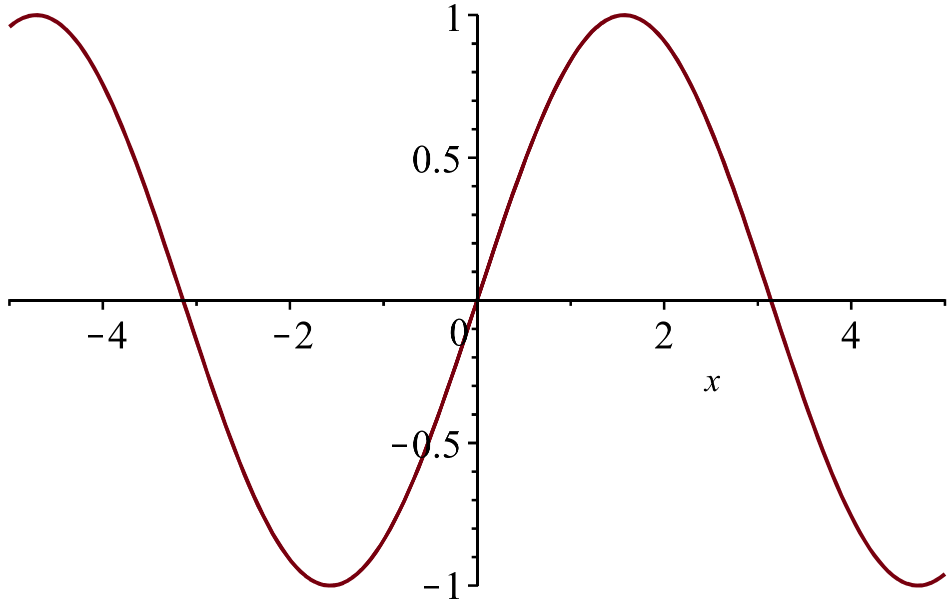
Exercise 3.2

```
> simplify(2*x/(x^2-1)+1/(x+x^2)+1/(x-x^2));
```
$$\frac{2}{x} \tag{25}$$

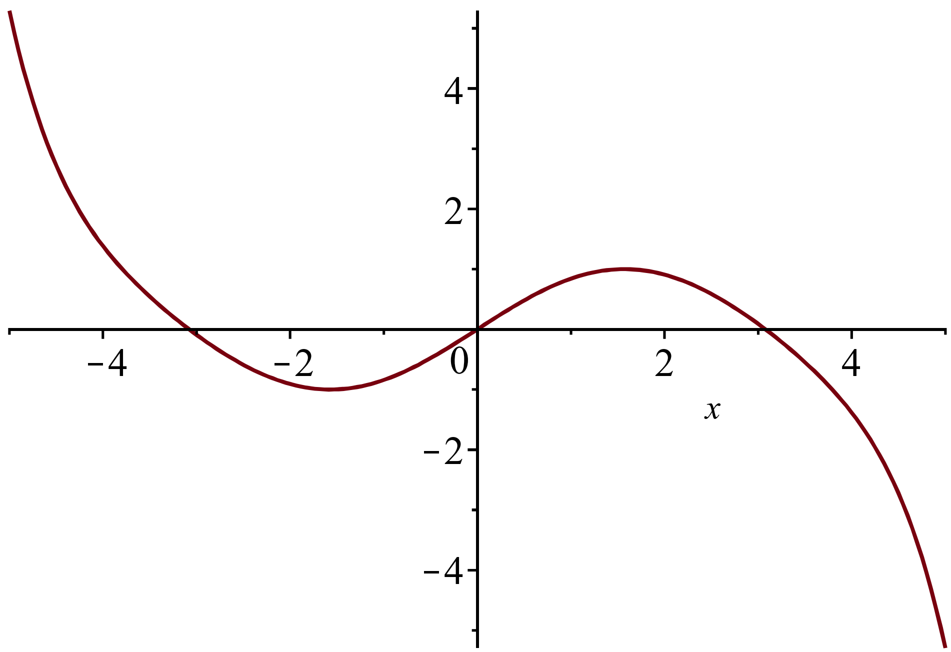
To do this by hand, note that $x^2 - 1 = (x+1)(x-1)$ and $x + x^2 = x\,(x+1)$ and $x - x^2 = -x(x-1)$. Thus, our expression is $\frac{2\,x^2}{x\,(x+1)\,(x-1)} + \frac{x-1}{x\,(x+1)\,(x-1)} - \frac{x+1}{x\,(x+1)\,(x-1)}$, which simplifies to $\frac{2\,x^2 - 2}{x\,(x+1)\,(x-1)}$. The numerator here factors as $2\,(x+1)\,(x-1)$ and so everything cancels to leave $\frac{2}{x}$.

Exercise 4.1

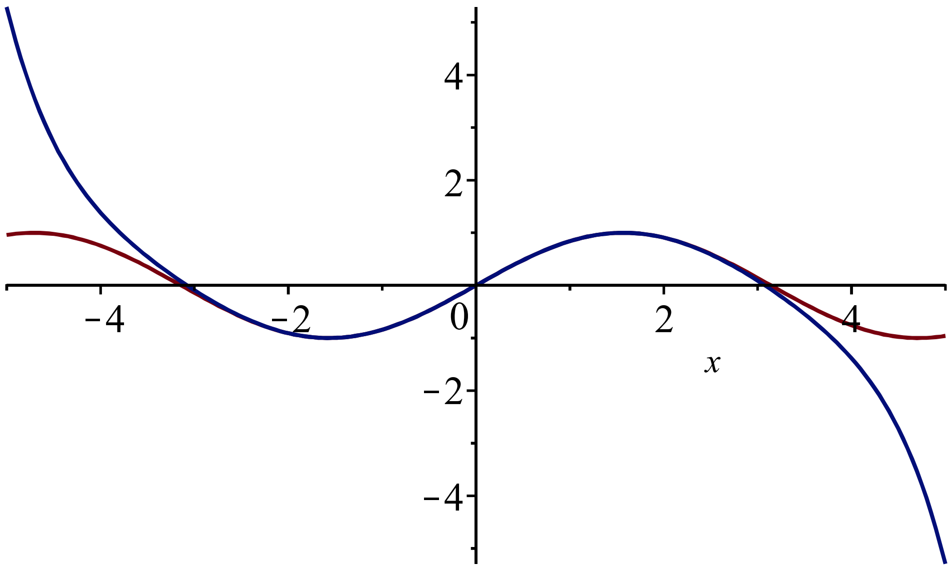
```
> plot(sin(x),x=-5..5);
```




```
> plot(x-x^3/6+x^5/120-x^7/5040,x=-5..5);
```




```
> plot([sin(x),x-x^3/6+x^5/120-x^7/5040],x=-5..5);
```



The two graphs are very close together for x between about -3 and 3, but outside that range they move apart very rapidly.

Exercise 5.1

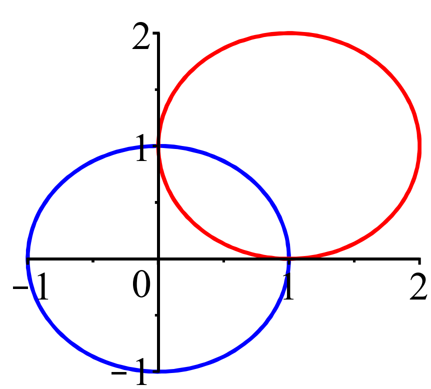
```
> solve({x^2+y^2=1,(x-1)^2+(y-1)^2=1},{x,y});
```

$$\{x=1, y=0\}, \{x=0, y=1\} \qquad (26)$$

To obtain this by hand, expand out the second equation to get $x^2 - 2x + 1 + y^2 - 2y + 1 = 1$, or equivalently $x^2 + y^2 = 2x + 2y - 1$. Subtracting the first equation gives $0 = 2x + 2y - 2$, so $x + y = 1$. Squaring this gives $x^2 + y^2 + 2xy = 1$, and subtracting the first equation gives $2xy = 0$. This means that either x or y must be zero, and the equation $x + y = 1$ means that the other one must be 1.

Geometrically, the equation $x^2 + y^2 = 1$ describes the circle of radius one centred at the origin, and the equation $(x-1)^2 + (y-1)^2 = 1$ describes the circle of radius one centred at (1,1). The two circles intersect at the points (1,0) and (0,1), which give the two solutions to our equations.

```
> plots[display](
    plot([cos(t),sin(t),t=0..2*Pi],color=blue),
    plot([1+cos(t),1+sin(t),t=0..2*Pi],color=red)
  );
```



Exercise 6.1

```
> diff(ln(ln(ln(x))),x);
```

$$\frac{1}{x \ln(x) \ln(\ln(x))} \quad (27)$$

```
> (3*x+4)/(2*x+3);
```

$$\frac{3x+4}{2x+3} \quad (28)$$

```
> diff(%,x);
```

$$\frac{3}{2x+3} - \frac{2(3x+4)}{(2x+3)^2} \quad (29)$$

```
> simplify(%);
```

$$\frac{1}{(2x+3)^2} \quad (30)$$

```
> (1+x^2+x^4/2)*exp(-x^2);
```

$$\left(1 + x^2 + \frac{1}{2} x^4\right) e^{-x^2} \quad (31)$$

```
> diff(%,x);
```

$$\left(2x^3 + 2x\right) e^{-x^2} - 2\left(1 + x^2 + \frac{1}{2} x^4\right) x e^{-x^2} \quad (32)$$

```
> simplify(%);
```

$$-x^5 e^{-x^2} \quad (33)$$

Exercise 6.2

```
> int(sqrt(1-x^2),x=0..1);
```

$$\frac{\pi}{4} \quad (34)$$