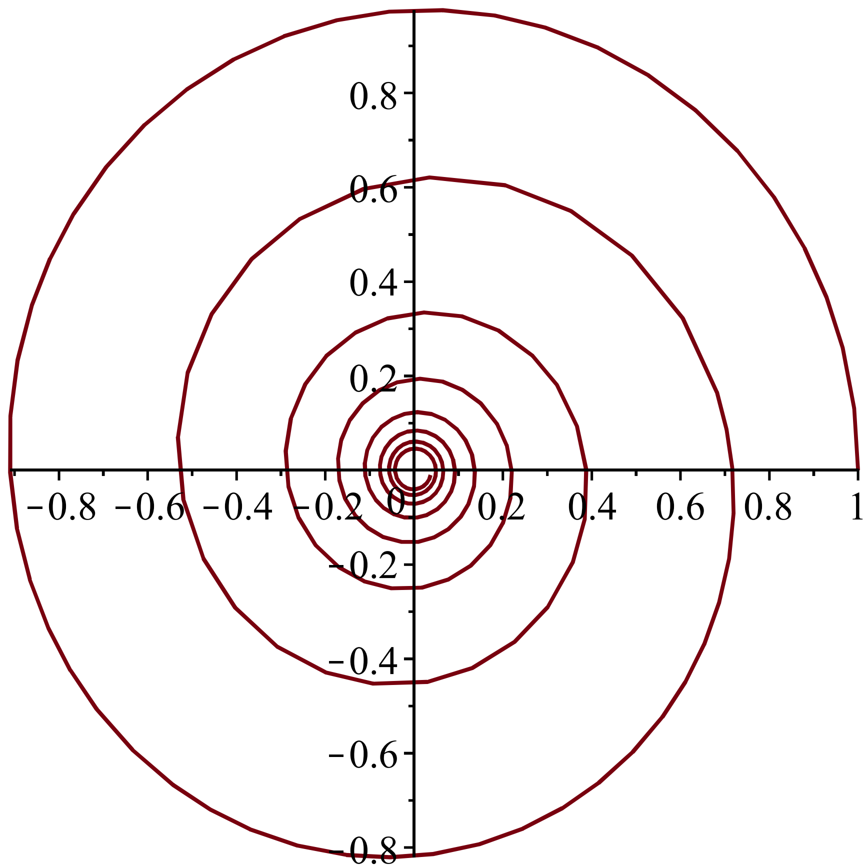
More plotting

Exercise 1.1

```
> restart;
```

(a)

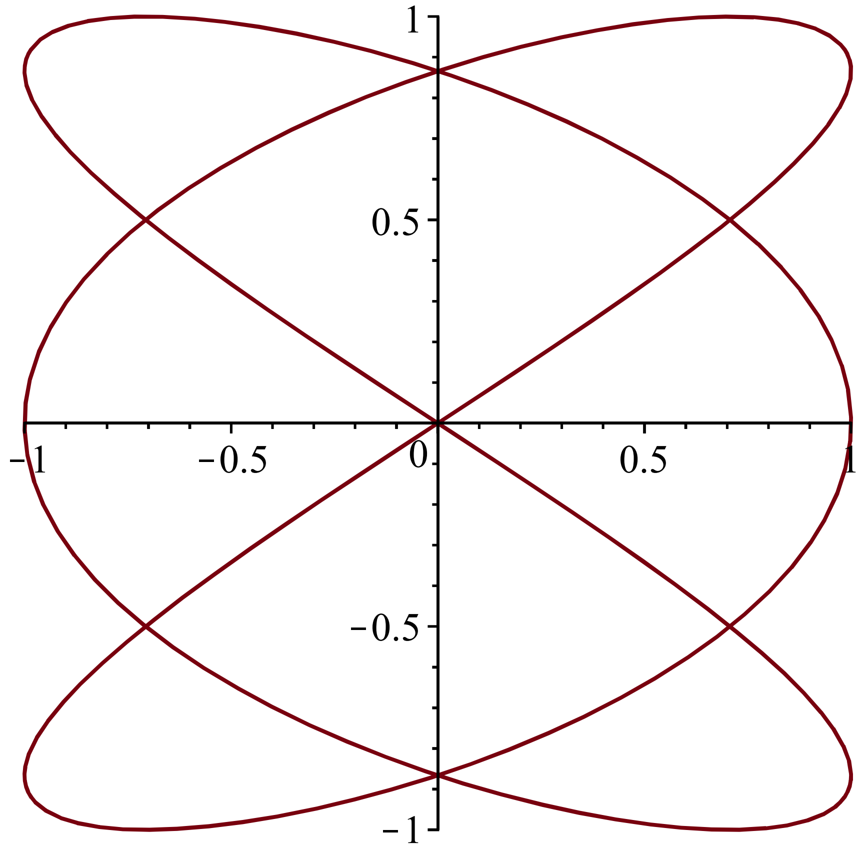
```
> plot([cos(10*t)/(1+t^2),sin(10*t)/(1+t^2),t=0..5]);
```

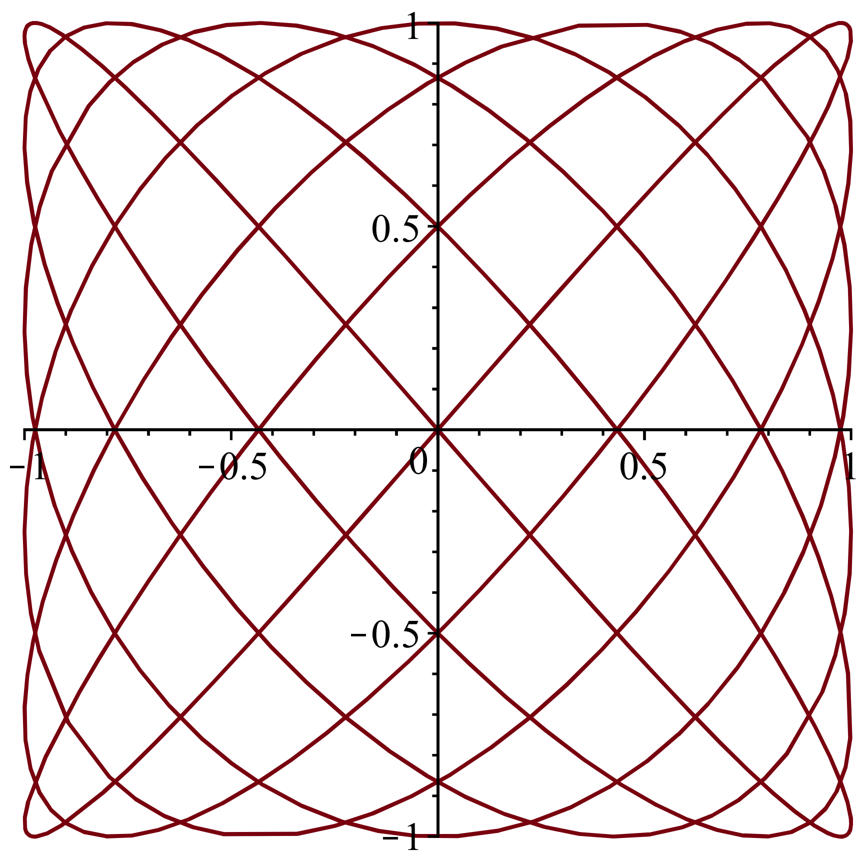


This starts at $(1, 0)$ and moves anticlockwise spiralling in towards the origin.

(b)

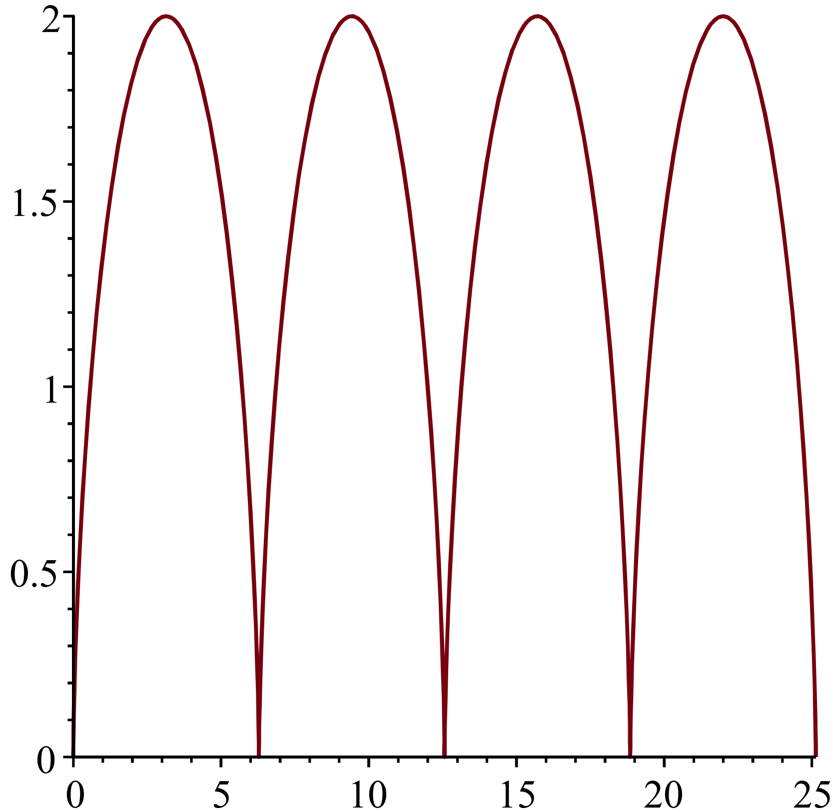
```
> plot([sin(3*t),sin(2*t),t=0..2*Pi]);
  plot([sin(6*t),sin(7*t),t=0..2*Pi]);
```


1
0.5
−1
−0.5
0
0.5
1
−0.5
−1

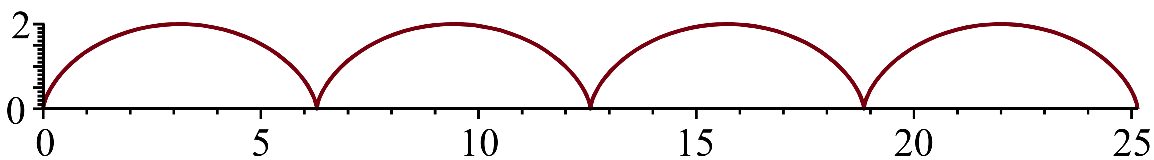

1
0.5
−1
−0.5
0
0.5
1
−0.5
−1

(c)

```
> plot([t-sin(t),1-cos(t),t=0..8*Pi]);
```

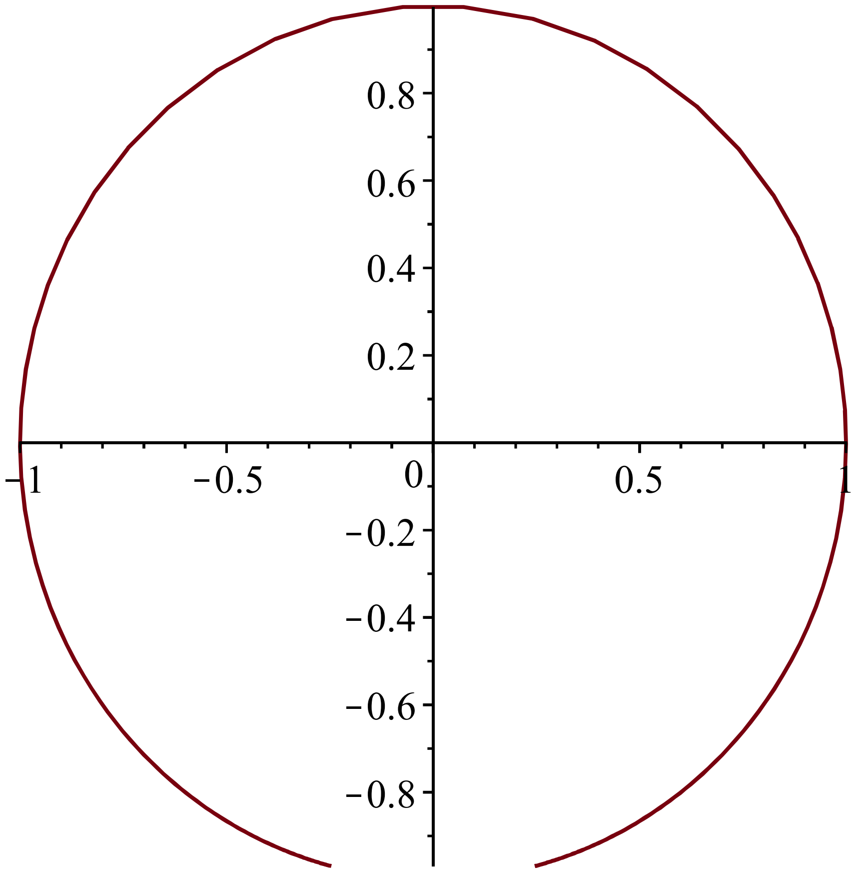



```
> plot([t-sin(t),1-cos(t),t=0..8*Pi],scaling=constrained);
```



(d)

```
> plot([2*t/(1+t^2),(1-t^2)/(1+t^2),t=-8..8]);
```



This is (most of) a circle of radius one, centred at the origin. To see algebraically that the point $\left(\frac{2\,t}{1+t^2}, \frac{1-t^2}{1+t^2}\right)$ lies on the circle, we must show that $\left(\frac{2\,t}{1+t^2}\right)^2 + \left(\frac{1-t^2}{1+t^2}\right)^2 = 1$:

```
> simplify((2*t/(1+t^2))^2+((1-t^2)/(1+t^2))^2);
```

$$1 \tag{1}$$

To do this by hand, note that

$$\left(1-t^2\right)^2 + (2\,t)^2 = 1 - 2\,t^2 + t^4 + 4\,t^2 = 1 + 2\,t^2 + t^4 = \left(1+t^2\right)^2.$$

Divide this by $\left(1+t^2\right)^2$ to get the required result.

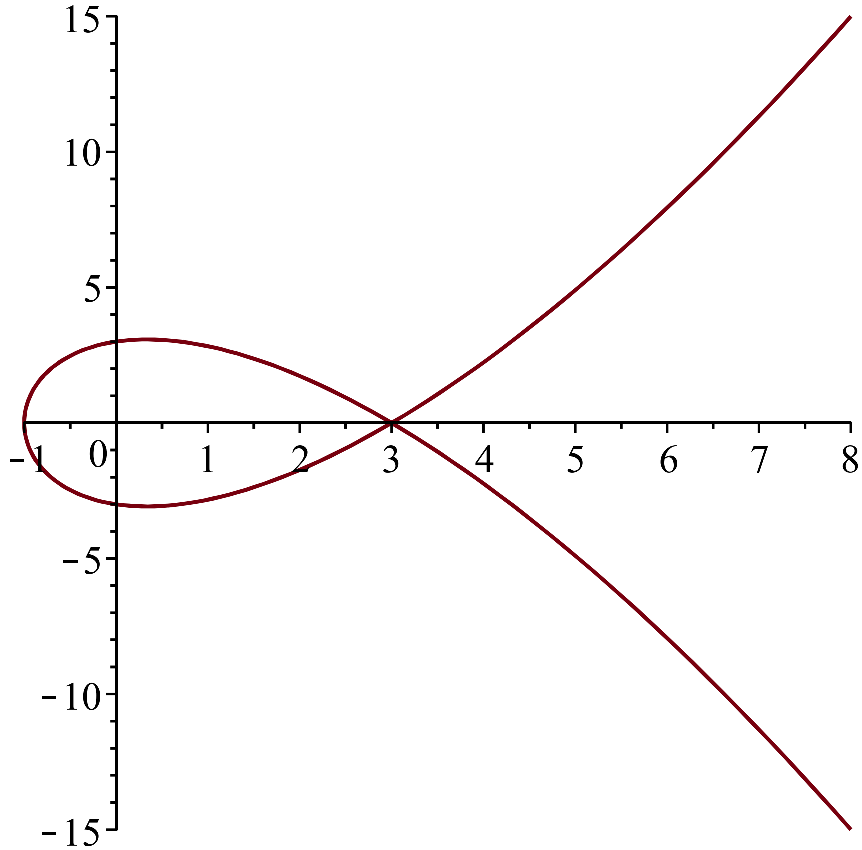
Exercise 1.2

```
> restart;
> x := 4*t^2-1;
  y := 8*t^3-8*t;
```

$$x := 4\,t^2 - 1$$
$$y := 8\,t^3 - 8\,t \tag{2}$$

(a)

```
> plot([x,y,t=-1.5..1.5]);
```



(b) We can see from the graph that the curve crosses itself at the point where $x = 3$ and $y = 0$. We can find the corresponding values of t as follows:

```
> solve({x=3,y=0},{t});
```

$$\{t = 1\}, \{t = -1\} \quad (3)$$

We can substitute back to check that this is correct:

```
> subs(t=1,[x,y]);
```

$$[3, 0] \quad (4)$$

```
> subs(t=1,[x,y]);
```

$$[3, 0] \quad (5)$$

(c)

```
> solve(x=0,{t});
```

$$\left\{t = \frac{1}{2}\right\}, \left\{t = -\frac{1}{2}\right\} \quad (6)$$

```
> subs(t=1/2,[x,y]);
```

$$[0, -3] \quad (7)$$

```
> subs(t=-1/2,[x,y]);
```

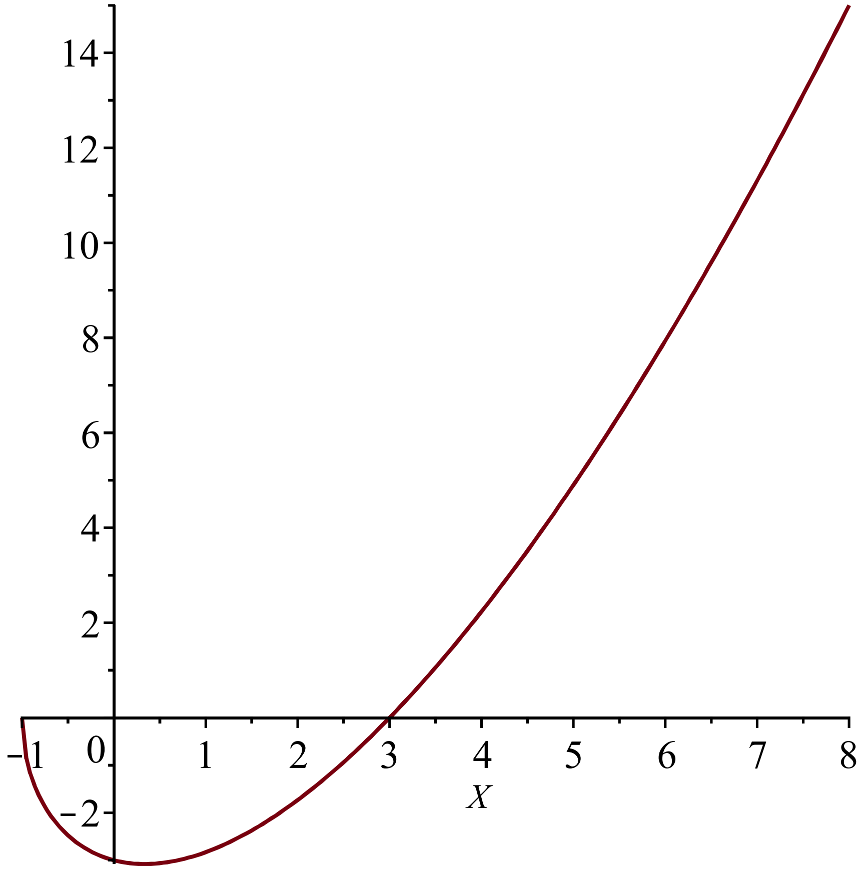
$$[0, 3] \quad (8)$$

We conclude that the curve crosses the y axis at the points $[0, -3]$ (when $t = \frac{1}{2}$) and $[0, 3]$ (when

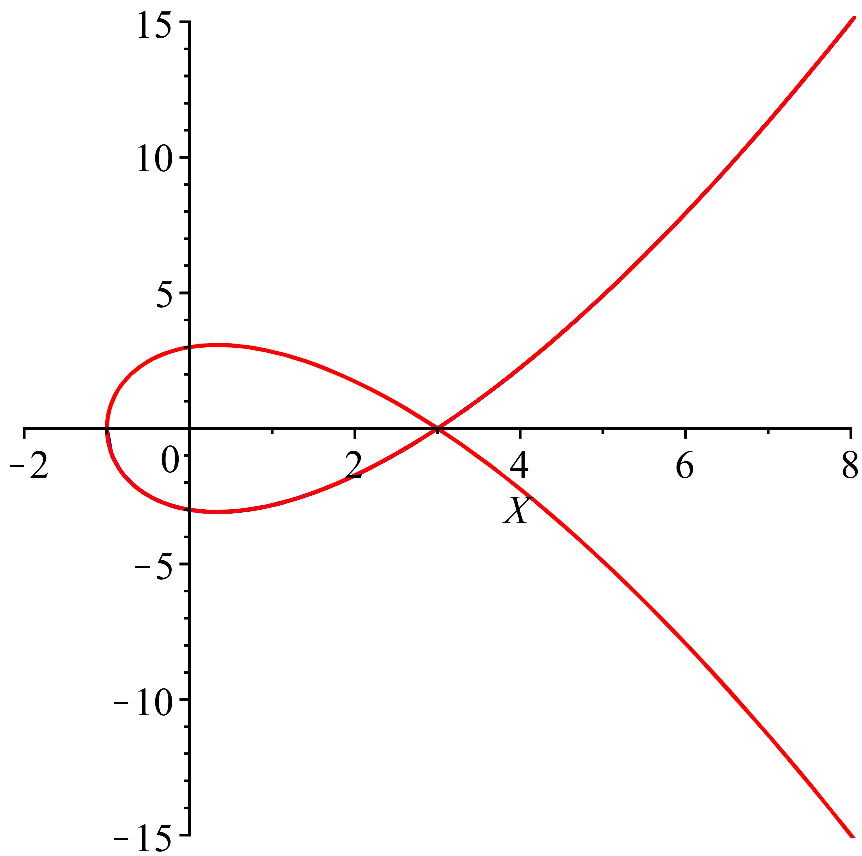
$t = -\frac{1}{2}$).

(d)

```
> plot((X-3)*sqrt(X+1),X=-1..8);
```




```
> plots[display](
      plot((X-3)*sqrt(X+1),X=-1..8,color=blue),
      plot([x,y,t=-2..2],color=red),
      view=[-2..8,-15..15]
  );
```



The two curves fit together, indicating that our functions $x = 4t^2 - 1$ and $y = 8t^3 - 8t$ satisfy $y = (x-3)\sqrt{x+1}$. As the graph of $(X-3)\sqrt{X+1}$ covers only half of the curve, there must be some wrinkle in the story somewhere, and it is not too hard to see that this comes from the sign ambiguity in the square root. The right thing to check is the equation obtained by squaring, which says that $y^2 = (x-3)^2(x+1)$. Maple tells us that this is indeed correct:

```
> simplify(y^2-(x-3)^2*(x+1));
```

$$0 \quad \textbf{(9)}$$

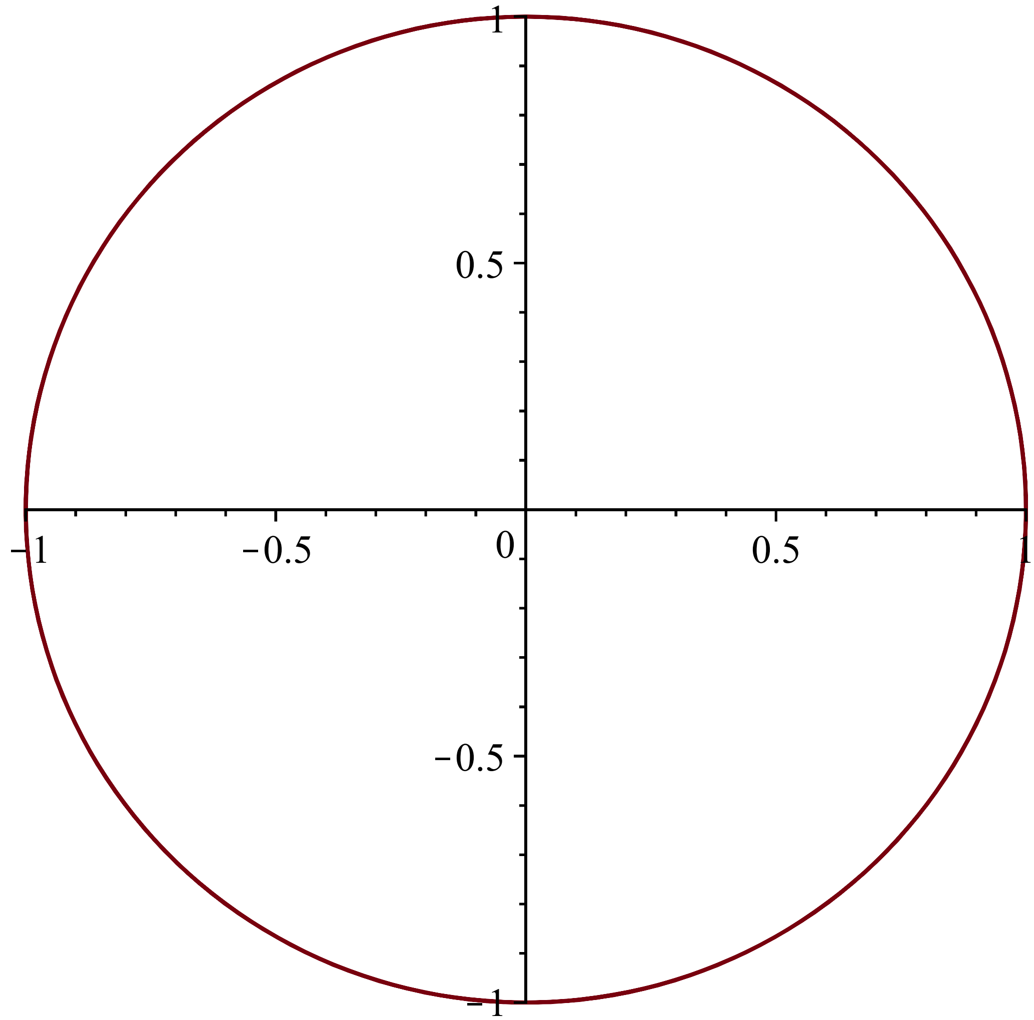
Exercise 1.3

```
> restart;
```

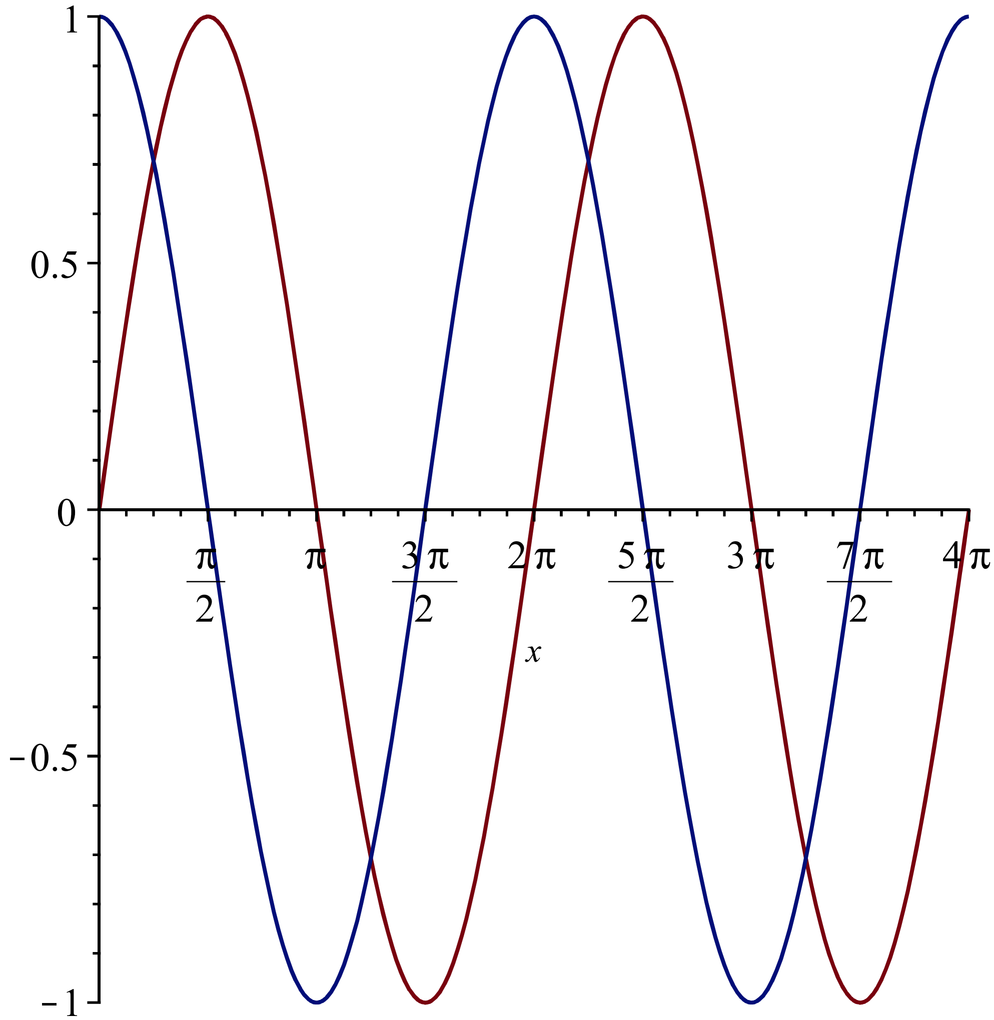
In the command `plot([sin(x),cos(x),x=0..4*Pi])`, the range `x=0..4*Pi` is inside the square brackets, so this is a parametric plot, with the horizontal coordinate being $\sin(x)$ and the vertical coordinate being $\cos(x)$. This is just a circle of radius one around the origin.

Error, missing operator or `;`

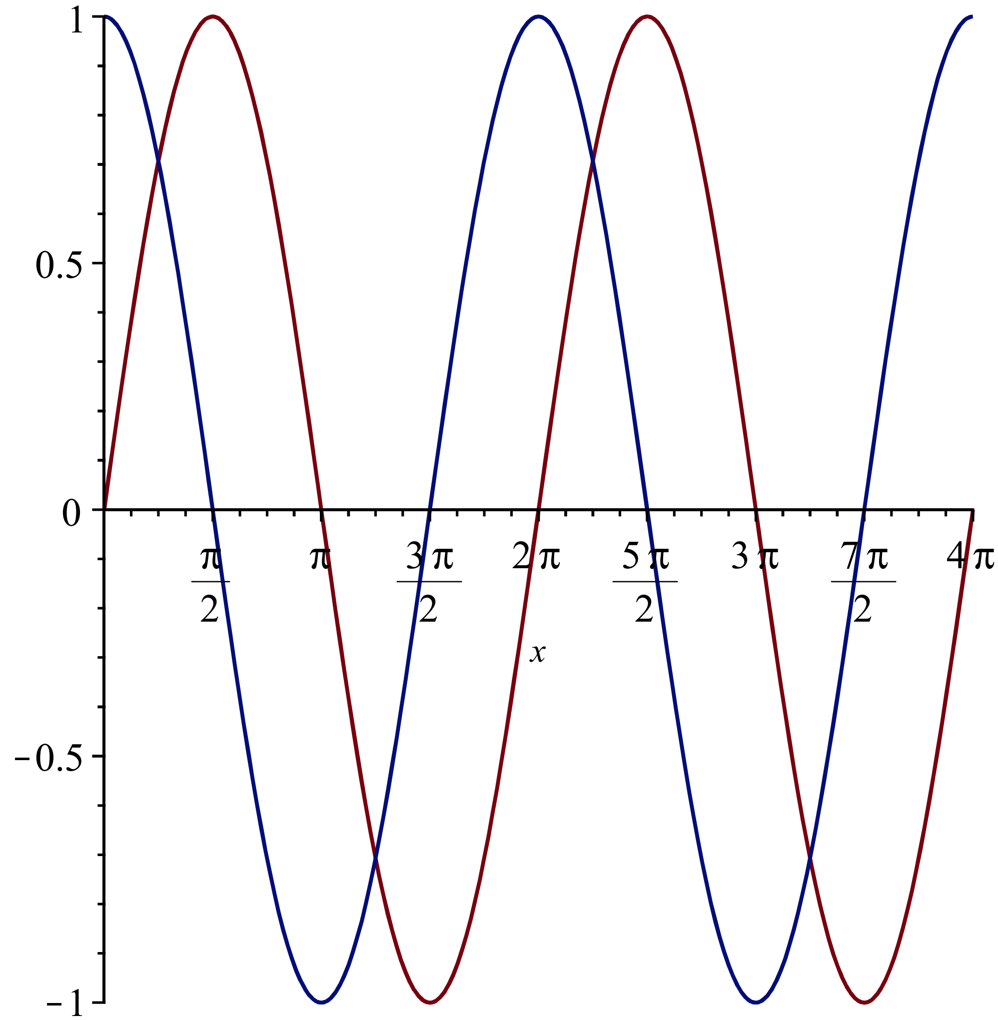
```
> plot([sin(x),cos(x),x=0..4*Pi]);
```



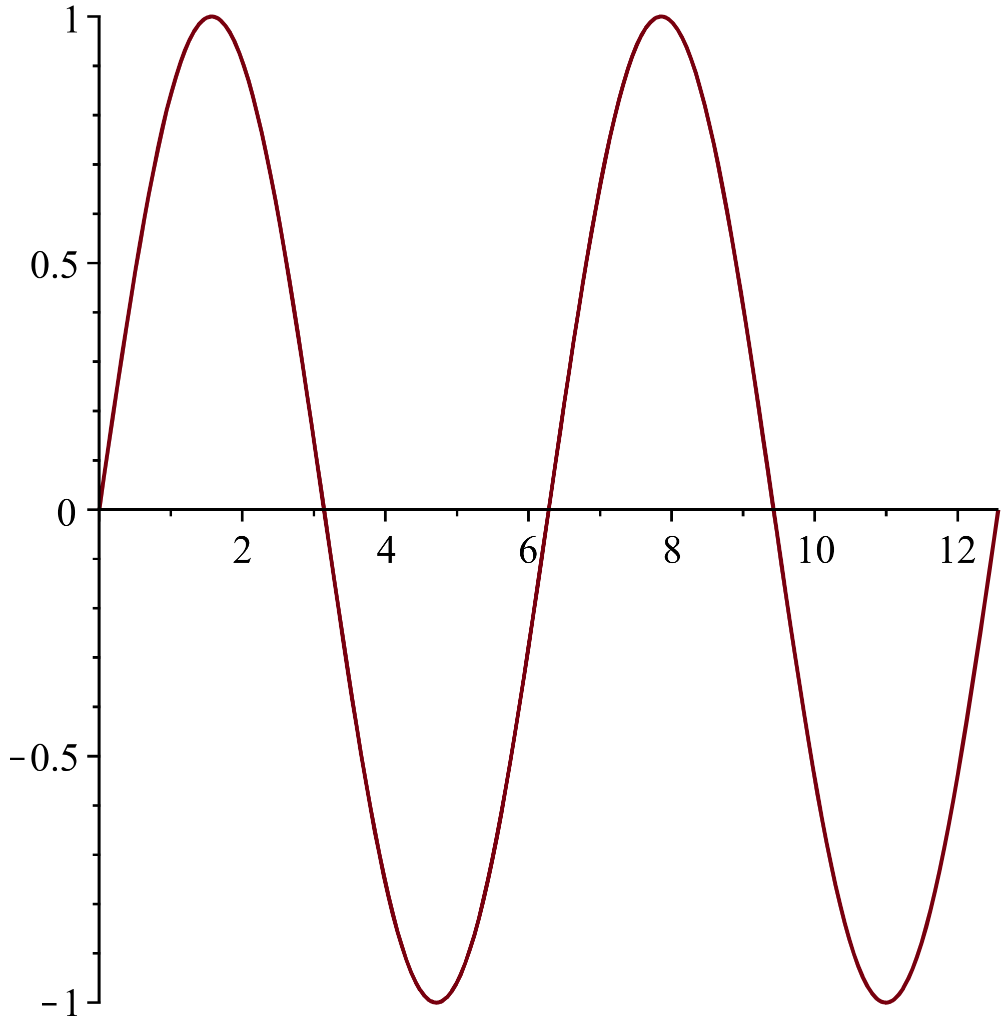
In the command **plot([sin(x),cos(x)],x=0..4*Pi)**, the range is outside the square brackets. We therefore have an ordinary plot of two different functions shown together, the first (in red) being $\sin(x)$, and the second (in green) being $\cos(x)$.



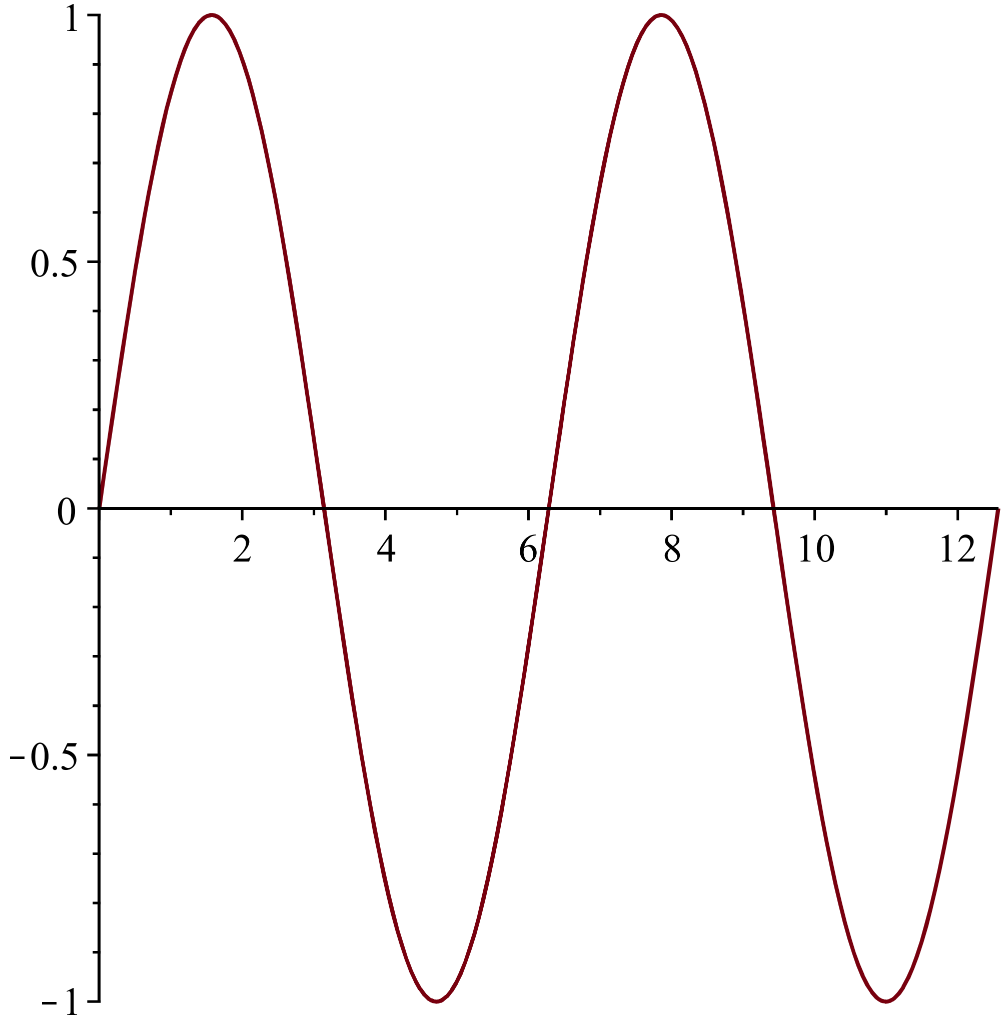

```
> plot([sin(x),cos(x)],x=0..4*Pi);
```



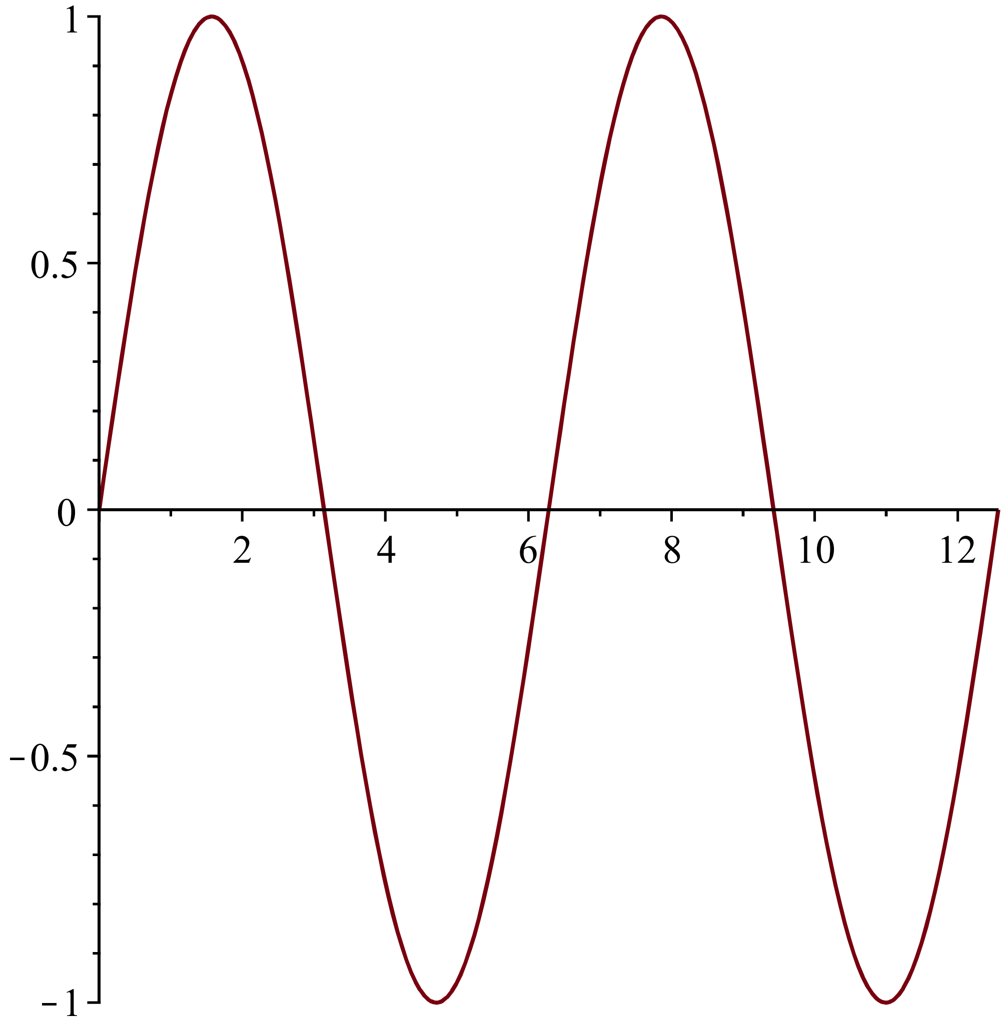
The command **plot([x,sin(x),x=0..4*Pi])** again has the range inside the square brackets, so it is a parametric plot of the curve $(x, \sin(x))$. However, because the horizontal coordinate is just x, this is the same as the ordinary graph of the function $\sin(x)$.



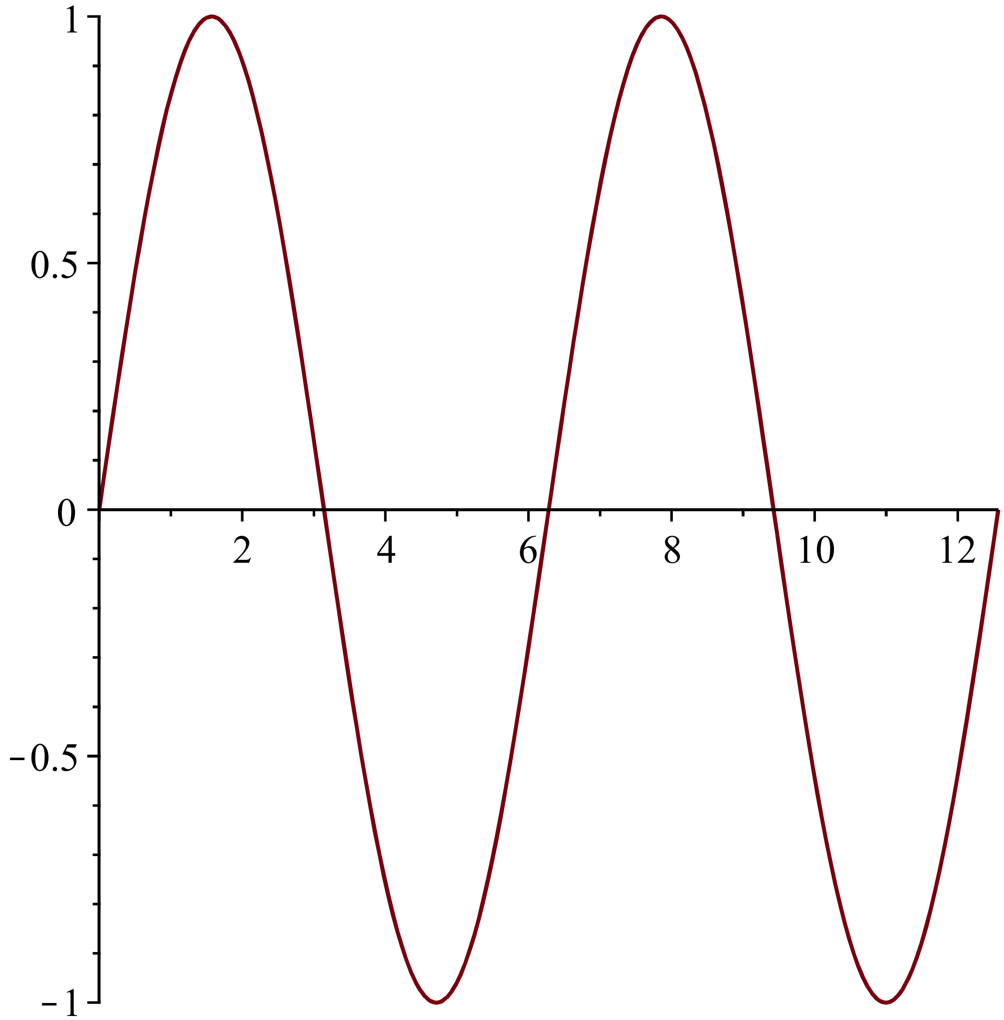

```
> plot([x,sin(x),x=0..4*Pi]);
```



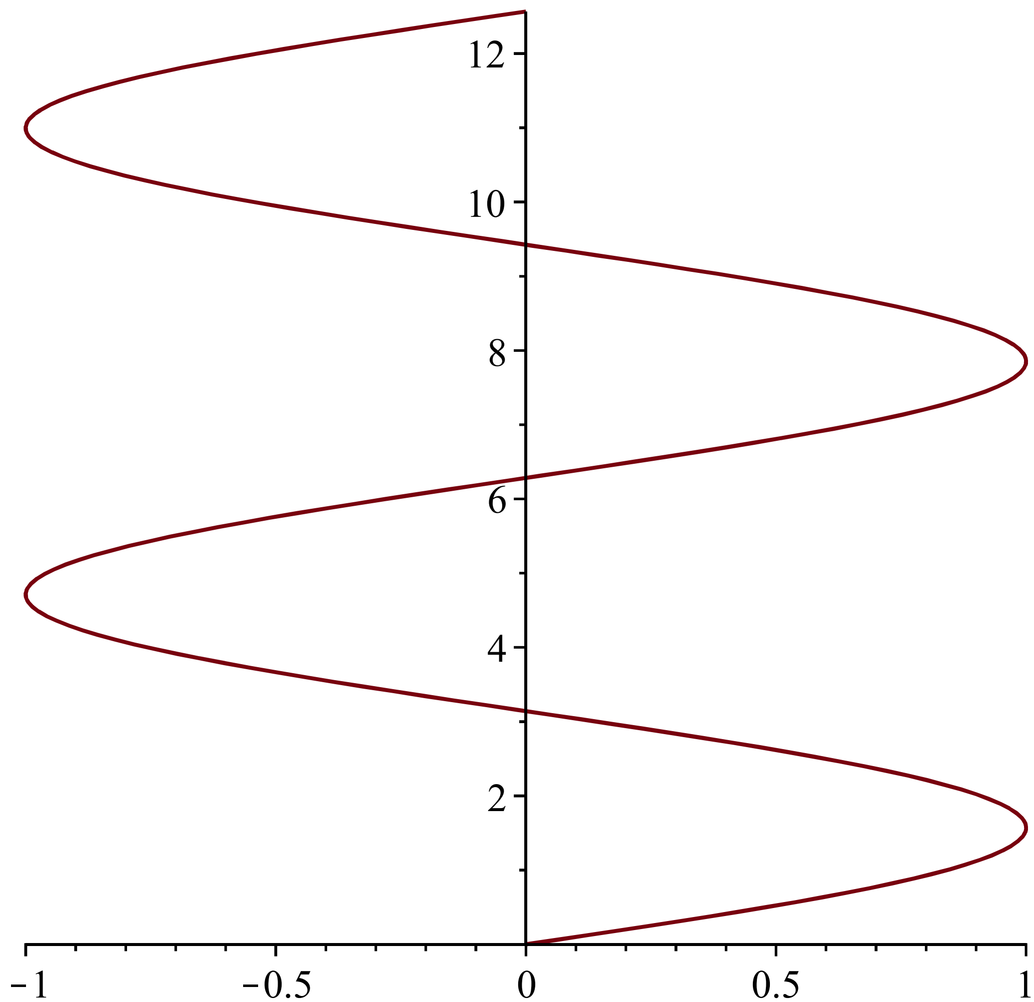
The command **`plot([y,sin(y),y=0..4*Pi])`** generates exactly the same picture as the previous command. Maple pays no attention to the convention that x usually represents the horizontal coordinate, and y usually represents the vertical one. Instead, the first entry (which is y here) is always the horizontal coordinate, and the second entry (which is $\sin(y)$) is always the vertical coordinate. With this interpretation, it does not make any difference whether the variable is called x or y.



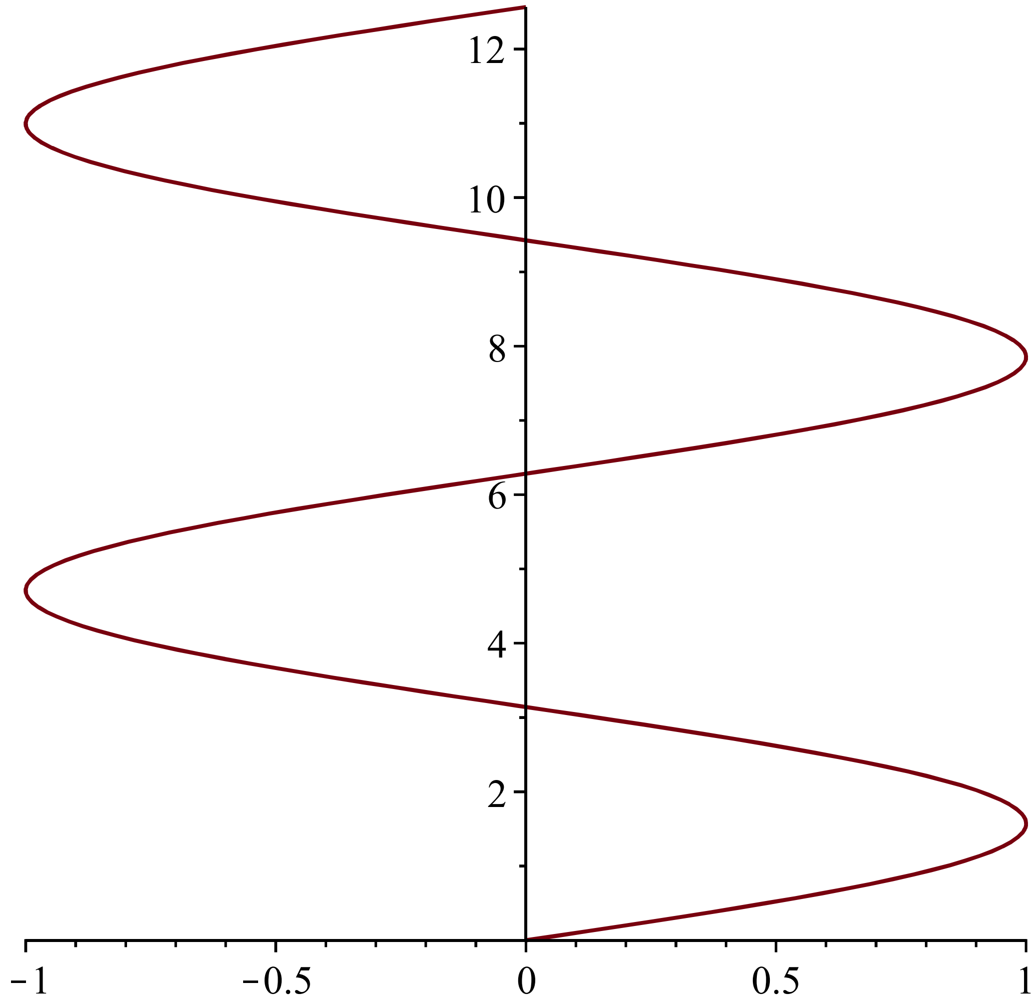

```
> plot([y,sin(y),y=0..4*Pi]);
```



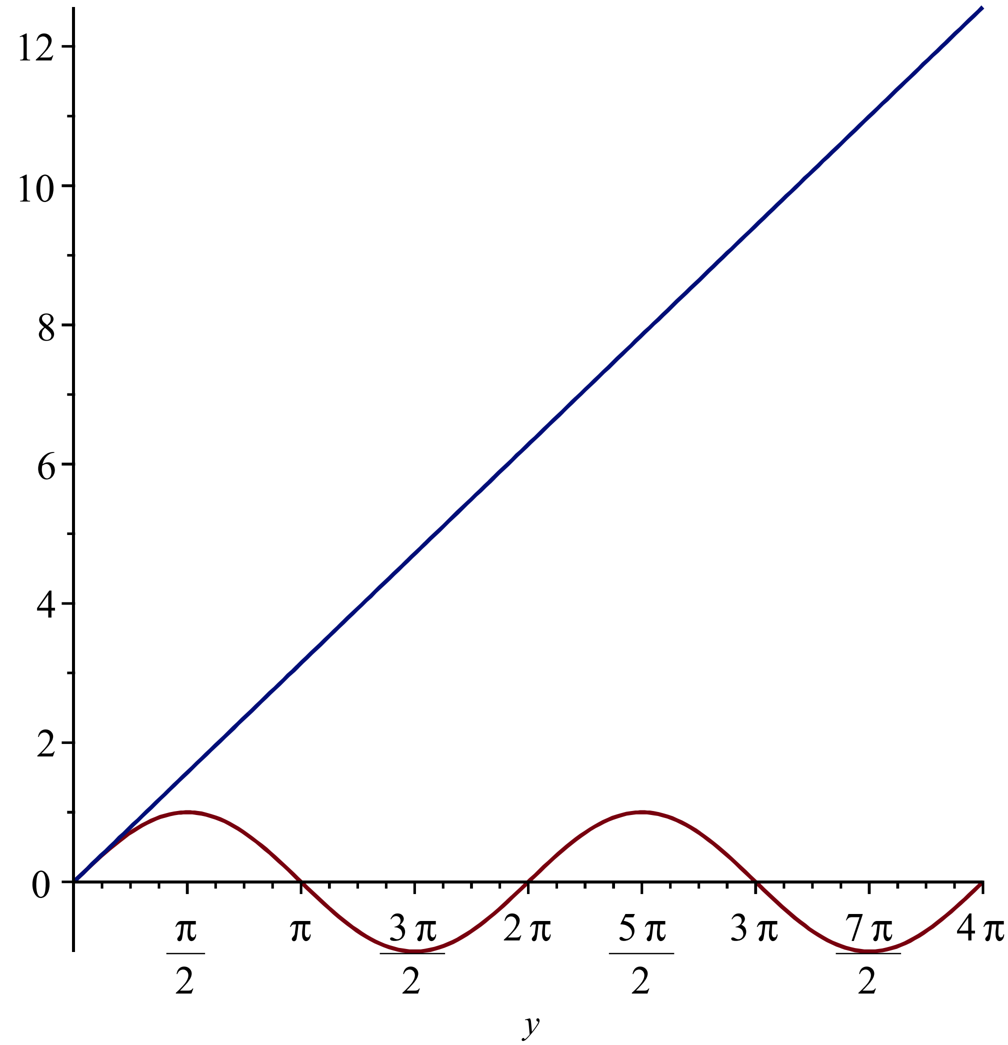
In the command **plot([sin(y),y,y=0..4*Pi])** the y comes second and so represents the vertical coordinate, and the function $\sin(y)$ gives the horizontal coordinate. We therefore get a standard sin wave running vertically.



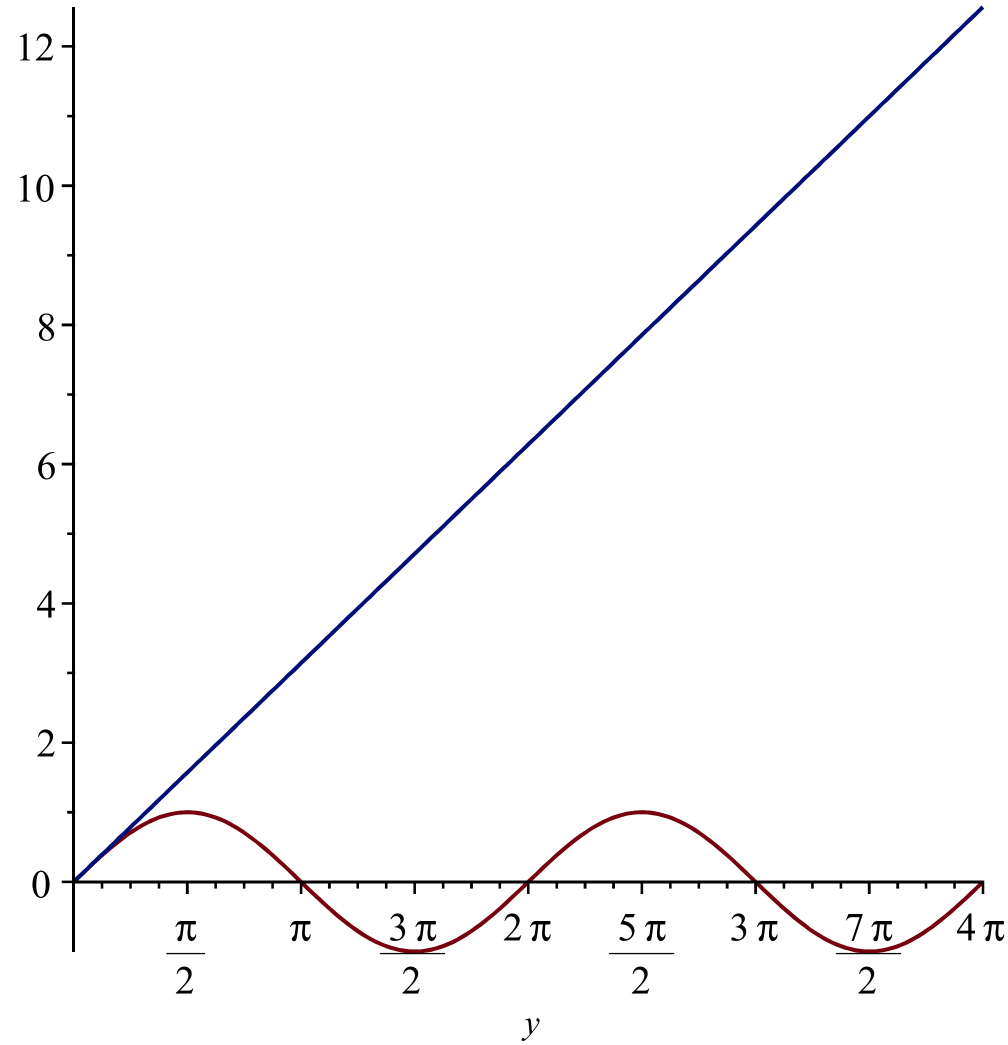

```
> plot([sin(y),y,y=0..4*Pi]);
```



Finally, in the command **plot([sin(y),y],y=0..4*Pi)** the range is again outside the square brackets, so we have an ordinary plot of the two functions $\sin(y)$ (in red) and y (in green) against y.




```
> plot([sin(y),y],y=0..4*Pi);
```



Exercise 2.1

```
> implicitplot(y^2=x^3-x,x=-2..2,y=-4..4);
```

$$implicitplot\left(y^2 = x^3 - x, x = -2..2, y = -4..4\right) \quad \textbf{(10)}$$

```
> implicitplot(y^2=x^3-x,x=-2..2,y=-4..4,grid=[100,100]);
```

$$implicitplot\left(y^2 = x^3 - x, x = -2..2, y = -4..4, grid = [100, 100]\right) \quad \textbf{(11)}$$

The picture below shows that the topology of the curve $y^2 = x^3 - x + a$ changes somewhere between $a = 0.3$ and $a = 0.4$.

```
> implicitplot([y^2=x^3-x+0.3,y^2=x^3-x+0.4],
               x=-2..2,y=-4..4,
               color=[red,blue],
               grid=[100,100]);
```

$$implicitplot\left(\left[y^2 = x^3 - x + 0.3, y^2 = x^3 - x + 0.4\right], x = -2..2, y = -4..4, color = [red, blue], grid = [100, 100]\right) \quad \textbf{(12)}$$

The following, more detailed picture, shows that the change is close to $a = 0.385$

```
> implicitplot([y^2=x^3-x+0.38,
                y^2=x^3-x+0.385,
                y^2=x^3-x+0.39],
               x=0.4..0.8,y=-0.5..0.5,
               color=[red,blue,green],
               grid=[100,100]);
```

$$implicitplot\left(\left[y^2 = x^3 - x + 0.38, y^2 = x^3 - x + 0.385, y^2 = x^3 - x + 0.39\right], x = 0.4..0.8, y = -0.5..0.5, color = [red, blue, green], grid = [100, 100]\right) \quad (13)$$

To find the exact value where the change occurs, note that we have $y = \sqrt{g(x)}$ or $y = -\sqrt{g(x)}$, where $g(x) = x^3 - x + a$. If $g(x)$ is negative near $x = 0.6$ then there is no square root and we have a gap in the graph. If $g(x)$ is positive near $x = 0.6$ then there are two separate square roots and so two disjoint branches of the graph. The crossover occurs when the graph of $g(x)$ just touches the x-axis near $x = 0.6$,

so we have a repeated root, where $g(x)$ and its derivative both vanish. The derivative is $3x^2 - 1$, and the

only place near $x = 0.6$ where this vanishes is at $x = \frac{1}{\sqrt{3}}$, which means that

$$g(x) = \left(\frac{1}{\sqrt{3}}\right)^3 - \frac{1}{\sqrt{3}} + a.$$

We want this to be zero as well, so we must have

$$a = \frac{1}{\sqrt{3}} - \left(\frac{1}{\sqrt{3}}\right)^3 = \frac{2}{3\sqrt{3}} = 0.3849001795$$

Exercise 2.2

We now look at the curve $x^2 + y^2 + a\,(\sin(2\,\mathrm{Pi}\,x) + \cos(2\,\mathrm{Pi}\,y)) = 100$. When $a = 0$, this is just a circle. When $a = 1$, it is a circle with wiggles:

```
> a := 1; implicitplot(x^2+y^2+a*(sin(2*Pi*x)+cos(2*Pi*y))=100,
               x=-11..11,y=-11..11,grid=[200,200]);
```

$$a := 1$$

$$implicitplot\left(x^2 + y^2 + \sin(2\pi x) + \cos(2\pi y) = 100, x = -11..11, y = -11..11, grid = [200, 200]\right) \quad (14)$$

When a is a little larger, some bubbles break away from the circle:

```
> a := 5; implicitplot(x^2+y^2+a*(sin(2*Pi*x)+cos(2*Pi*y))=100,
               x=-11..11,y=-11..11,grid=[200,200]);
```

$$a := 5$$

$$implicitplot\left(x^2 + y^2 + 5\sin(2\pi x) + 5\cos(2\pi y) = 100, x = -11..11, y = -11..11, grid = [200, 200]\right) \quad (15)$$

As a increases, we get an octagonal curve with squarish wiggles, with rows of bubbles running parallel to the curve, with the more distant rows being smaller.

```
> a := 20; implicitplot(x^2+y^2+a*(sin(2*Pi*x)+cos(2*Pi*y))=100,
               x=-11..11,y=-11..11,grid=[500,500]);
```

$$a := 20$$

$$implicitplot\left(x^2+y^2+20\sin(2\pi x)+20\cos(2\pi y)=100, x=-11..11, y=-11..11, grid=[500,500]\right) \quad \textbf{(16)}$$

```
> a := 80; implicitplot(x^2+y^2+a*(sin(2*Pi*x)+cos(2*Pi*y))=100,
               x=-21..21,y=-21..21,grid=[200,200]);
```

$$a := 80$$

$$implicitplot\left(x^2+y^2+80\sin(2\pi x)+80\cos(2\pi y)=100, x=-21..21, y=-21..21, grid=[200,200]\right) \quad \textbf{(17)}$$

Exercise 3.1

```
> restart;
```

(a)

```
> listplot([10,40,20,30,50,10,20],style=POINT);
```

listplot([10, 40, 20, 30, 50, 10, 20], *style* = *POINT*) **(18)**

```
> listplot([1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,
           19,18,17,16,15,14,13,12,11,10,9,8,7,6,5,4,3,2,1,
           2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20],
           style=POINT);
```

listplot([1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20], *style* = *POINT*) **(19)**

(b)

```
> listplot([10,40,20,30,50,10,20],style=POINT,symbol=CROSS);
```

listplot([10, 40, 20, 30, 50, 10, 20], *style* = *POINT*, *symbol* = *CROSS*) **(20)**

```
> listplot([10,40,20,30,50,10,20],style=POINT,symbol=BOX);
```

listplot([10, 40, 20, 30, 50, 10, 20], *style* = *POINT*, *symbol* = *BOX*) **(21)**

(c)

```
> listplot([10,40,20,30,50,10,20]);
```

listplot([10, 40, 20, 30, 50, 10, 20]) **(22)**

Exercise 3.2

```
> restart;
```

```
> with(plots):
```

(a)

```
> ithprime(1);
```

2 **(23)**

```
> ithprime(2);
```

3 **(24)**

```
> ithprime(3);
```

5 **(25)**

```
> ithprime(4);
```

7 **(26)**

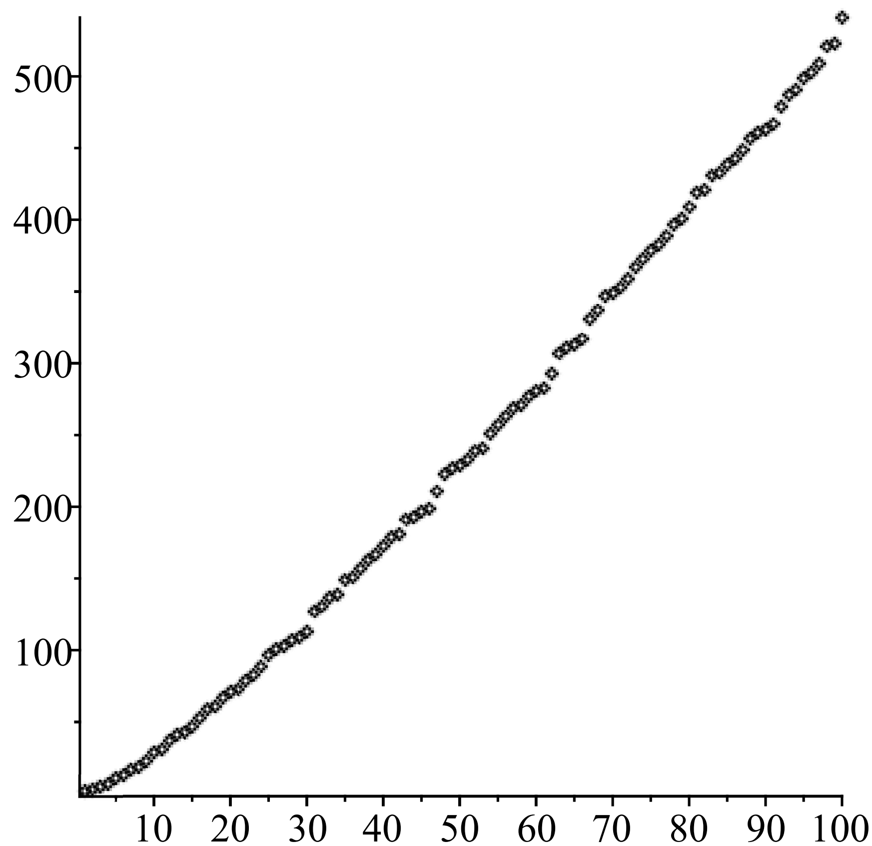
(b)

```
> seq(ithprime(i),i=1..100);
```

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199, 211, 223, 227, 229, 233, 239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 293, 307, 311, 313, 317, 331, 337, 347, 349, 353, 359, 367, 373, 379, 383, 389, 397, 401, 409, 419, 421, 431, 433, 439, 443, 449, 457, 461, 463, 467, 479, 487, 491, 499, 503, 509, 521, 523, 541 **(27)**

(c)

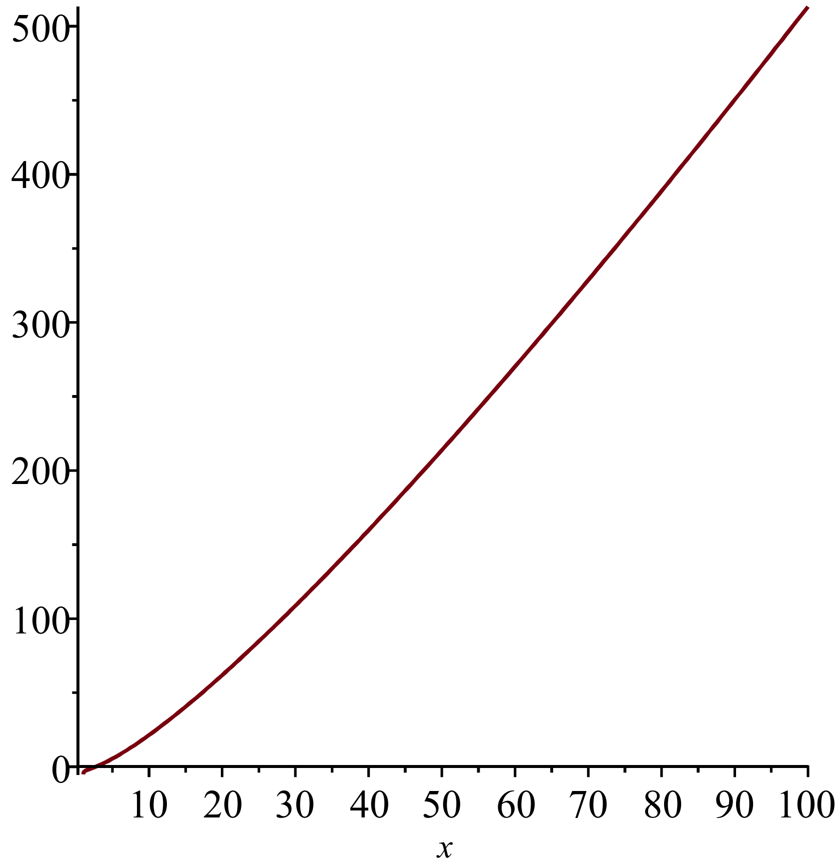
```
> listplot([seq(ithprime(n),n=1..100)],style=POINT);
```



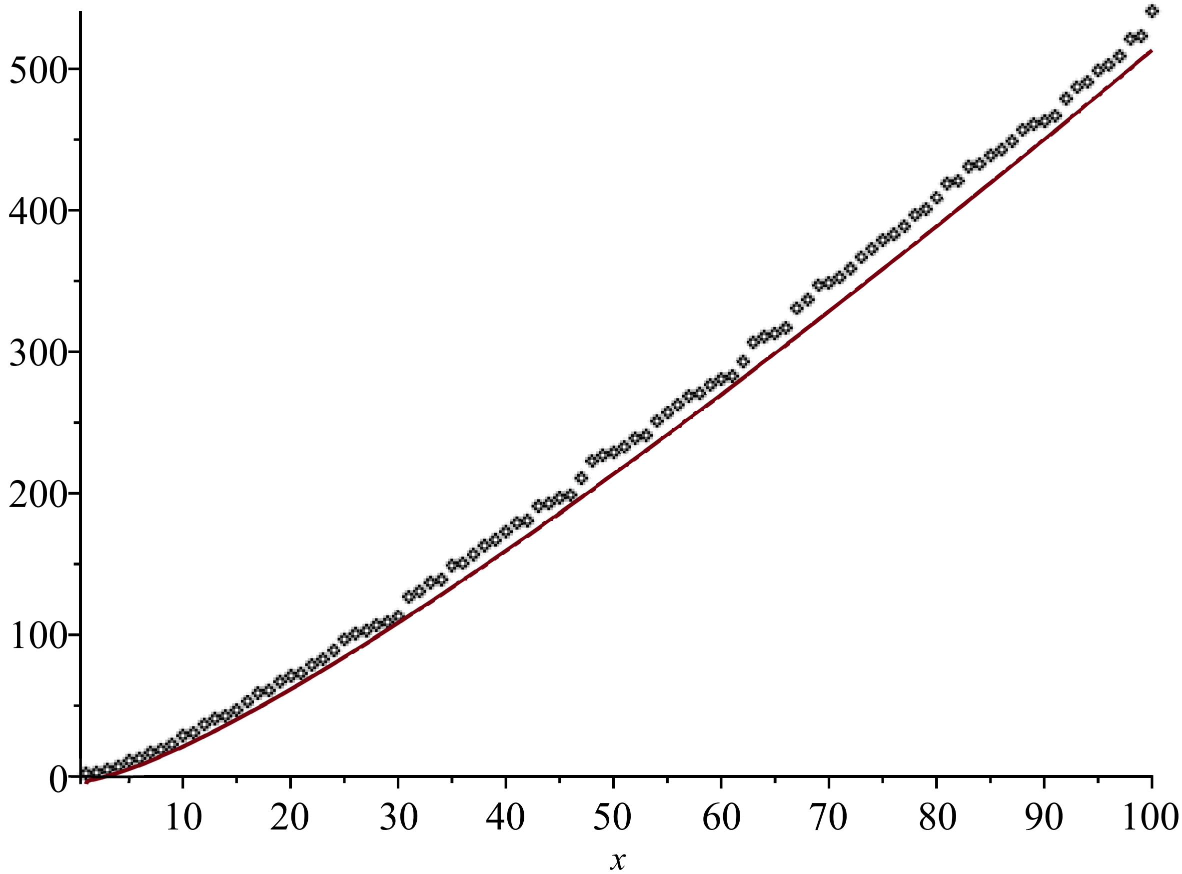
```
> pic1 := %:
```

(d)

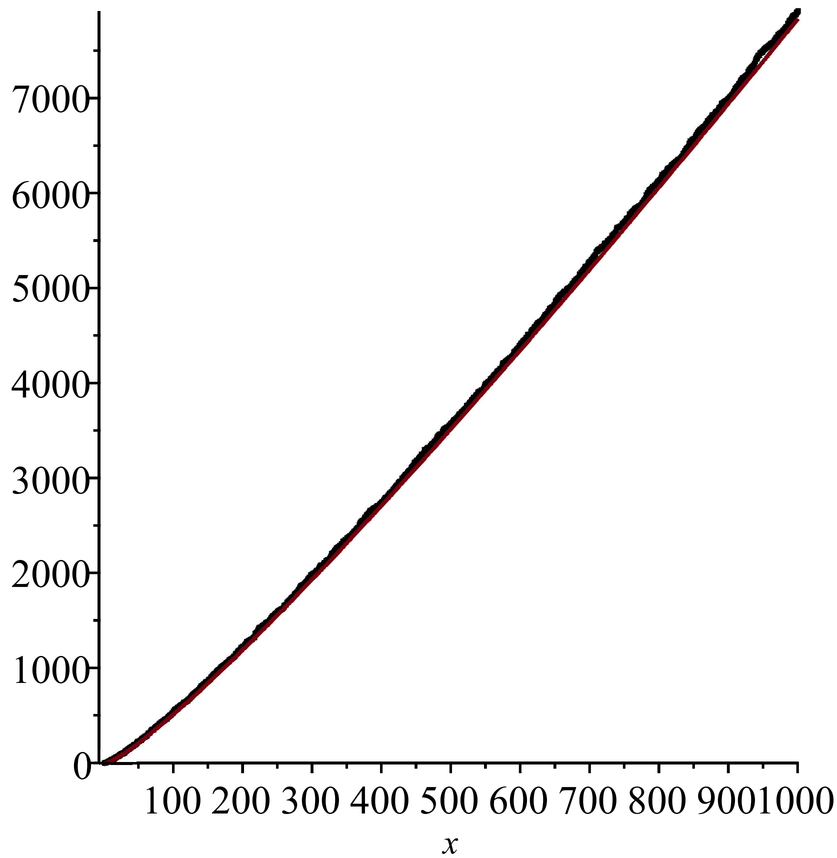
```
> plot(x*(ln(ln(x))+ln(x)-1),x=1..100);
```




```
> pic2:=%:
> display(pic1,pic2);
```




```
> display(
     listplot([seq(ithprime(i),i=1..1000)],
              style=POINT,symbol=POINT),
     plot(x*(ln(ln(x)) + ln(x) - 1),x=1..1000)
    );
```



Exercise 3.3

```
> restart;
> with(plots):
```

(a)

```
> 20!;
```

2432902008176640000 **(28)**

```
> evalf(20!);
```

$2.432902008\ 10^{18}$ **(29)**

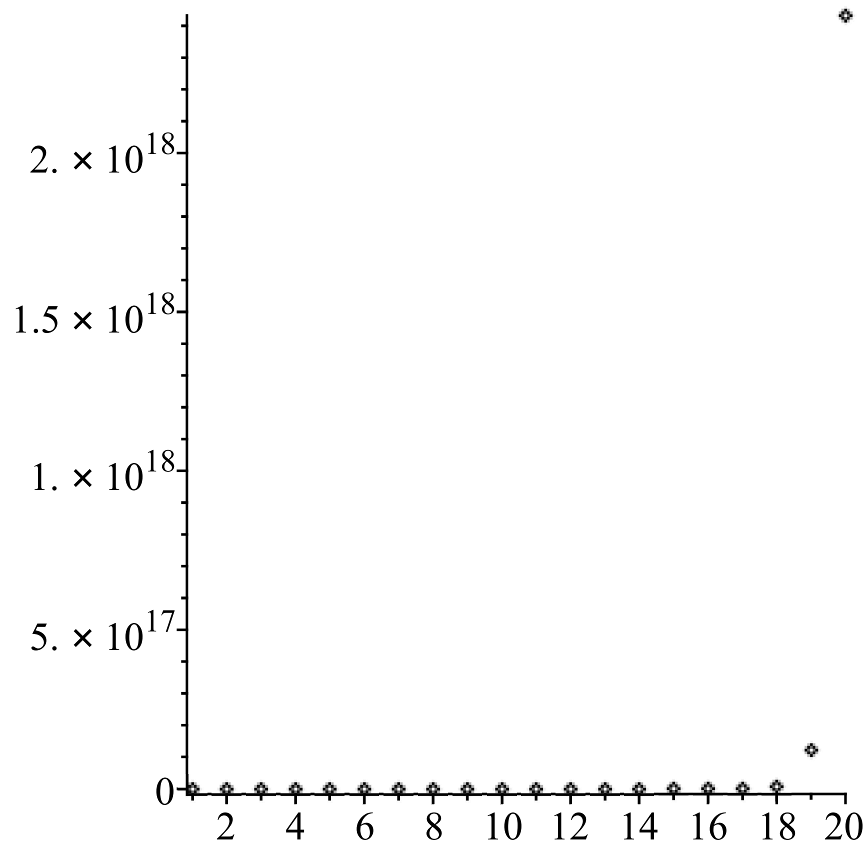
(b)

```
> seq(evalf(n!),n=1..20);
```

1., 2., 6., 24., 120., 720., 5040., 40320., 362880., $3.628800\ 10^{6}$, $3.9916800\ 10^{7}$, $4.79001600\ 10^{8}$, $6.227020800\ 10^{9}$, $8.717829120\ 10^{10}$, $1.307674368\ 10^{12}$, $2.092278989\ 10^{13}$, $3.556874281\ 10^{14}$, $6.402373706\ 10^{15}$, $1.216451004\ 10^{17}$, $2.432902008\ 10^{18}$ **(30)**

(c)

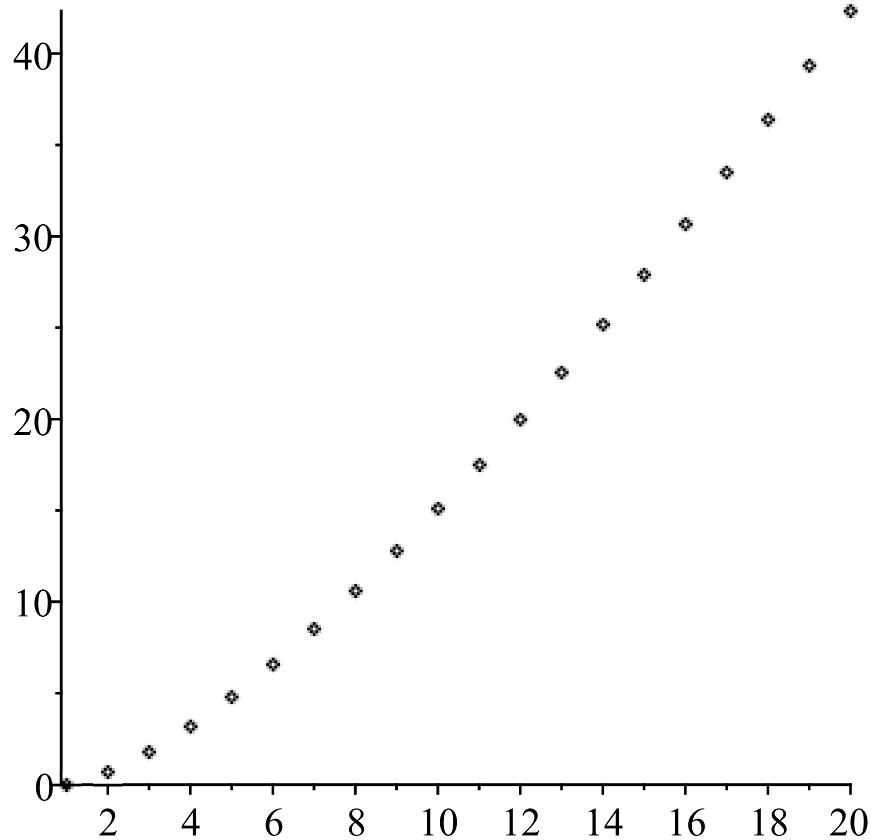
```
> listplot([seq(evalf(n!),n=1..20)],style=POINT,axes=FRAME);
```



We can see very little in this picture because 1!, 2!, ..., 17! are all very much smaller than 20!. The scale is chosen so that 20! fits in the picture, but this means that 1!, 2!, ..., 17! are jammed right down by the x-axis. (The effect of the option **axes=FRAME** is to draw the axis a little below $y=0$, allowing us to see the points. If we left out that option then the points would actually lie on the axis.)

(d)

```
> listplot([seq(log(n!),n=1..20)],style=POINT);
```



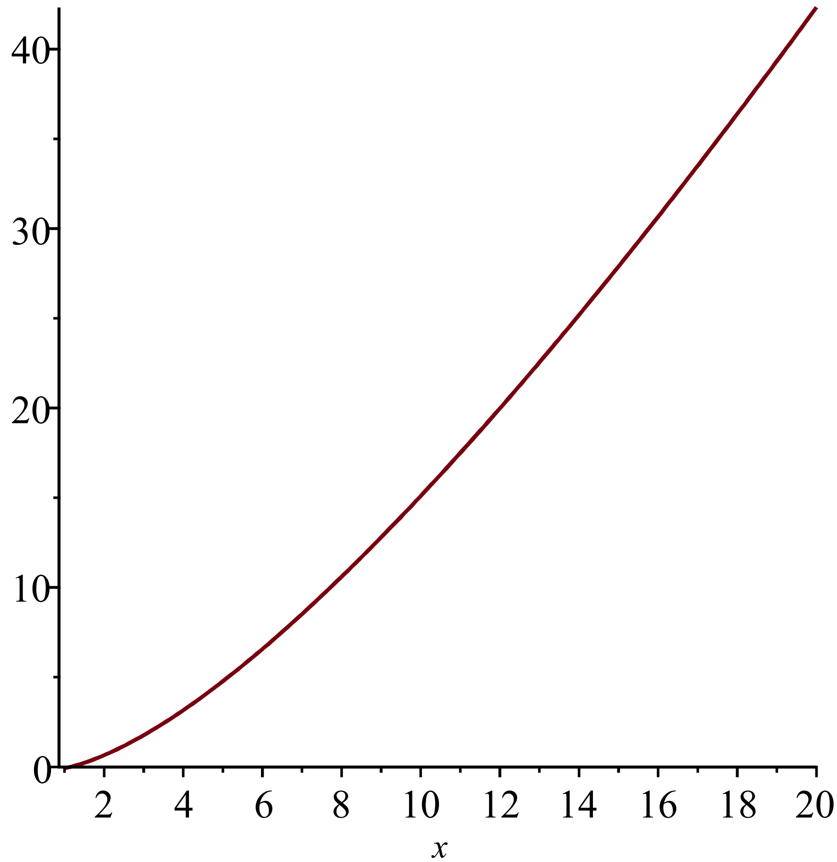

```
> pic1 :=%:
```

(e)

```
> f := (x) -> sqrt(2*Pi)*x^(x+1/2)*exp(-x);
```

$$f := x \mapsto \sqrt{2\,\pi}\;x^{x+\frac{1}{2}}\;e^{-x} \tag{31}$$

```
> plot(log(f(x)),x=1..20);
```




```
> pic2:=%:
> display(pic1,pic2);
```