# Maple revision

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## Exercise 1

```
> x[1] := sin(theta) * sin(phi);
```

$$x_1 := \sin(\theta)\,\sin(\phi) \quad (1)$$

```
> x[2] := sin(theta) * cos(phi);
```

$$x_2 := \sin(\theta)\,\cos(\phi) \quad (2)$$

```
> x[3] := cos(theta);
```

$$x_3 := \cos(\theta) \quad (3)$$

```
> simplify(x[1]^2 + x[2]^2 + x[3]^2);
```

$$1 \quad (4)$$

---

## Exercise 2

```
> u := 8-(x-y)^2*(x+y)^2*(16-4*x^2-y^4);
```

$$u := 8 - (x-y)^2\,(x+y)^2\,(16 - 4\,x^2 - y^4) \quad (5)$$

```
> v := subs( x = 2*cos(t), y = 2*sin(t), u);
```

$$v := 8 - (2\cos(t) - 2\sin(t))^2\,(2\cos(t) + 2\sin(t))^2\,(16 - 16\cos(t)^2 - 16\sin(t)^4) \quad (6)$$

```
> simplify(v);
```

$$8 + 1280\cos(t)^4 - 2048\cos(t)^6 - 256\cos(t)^2 + 1024\cos(t)^8 \quad (7)$$

```
> combine(v);
```

$$8\cos(8\,t) \quad (8)$$

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## Exercise 3

```
> solve({a*x+b*y+c*z=1,
         a*y+b*z+c*x=1,
         a*z+b*x+c*y=1},
        {x,y,z});
```

$$\left\{z = \frac{1}{a+b+c},\, y = \frac{1}{a+b+c},\, x = \frac{1}{a+b+c}\right\} \quad (9)$$

---

## Exercise 4

```
> fsolve(x^4+sin(x)=10^4,{x=10});
```

$$\{x = 10.00013603\} \quad (10)$$

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## Exercise 5

```
> f := (x) -> (exp(x) + x^7)/(exp(x) - x^7);
```

$$f := x \rightarrow \frac{e^x + x^7}{e^x - x^7} \quad (11)$$

```
> evalf(f(5));
```

$$-1.003806608 \quad (12)$$

```
> seq(  evalf(f(n)),  n = 0.. 50);
```

1., 2.163953414, -1.122527125, -1.018538376, -1.006687097, -1.003806608, **(13)**
-1.002886453, -1.002666759, -1.002846910, -1.003394057, -1.004415018,
-1.006163919, -1.009125826, -1.014201286, -1.023080202, -1.039011999,
-1.068473380, -1.125095655, -1.240266253, -1.498979413, -2.220795044,
-6.469168964, 5.574474436, 2.074081155, 1.418805147, 1.185231790, 1.085582359,
1.040109648, 1.018834822, 1.008814259, 1.004101418, 1.001896026, 1.000870654,
1.000397187, 1.000180057, 1.000081136, 1.000036354, 1.000016201, 1.000007184,
1.000003170, 1.000001392, 1.000000609, 1.000000266, 1.000000115, 1.000000050,
1.000000021, 1.000000009, 1.000000004, 1.000000002, 1.000000001, 1.000000000

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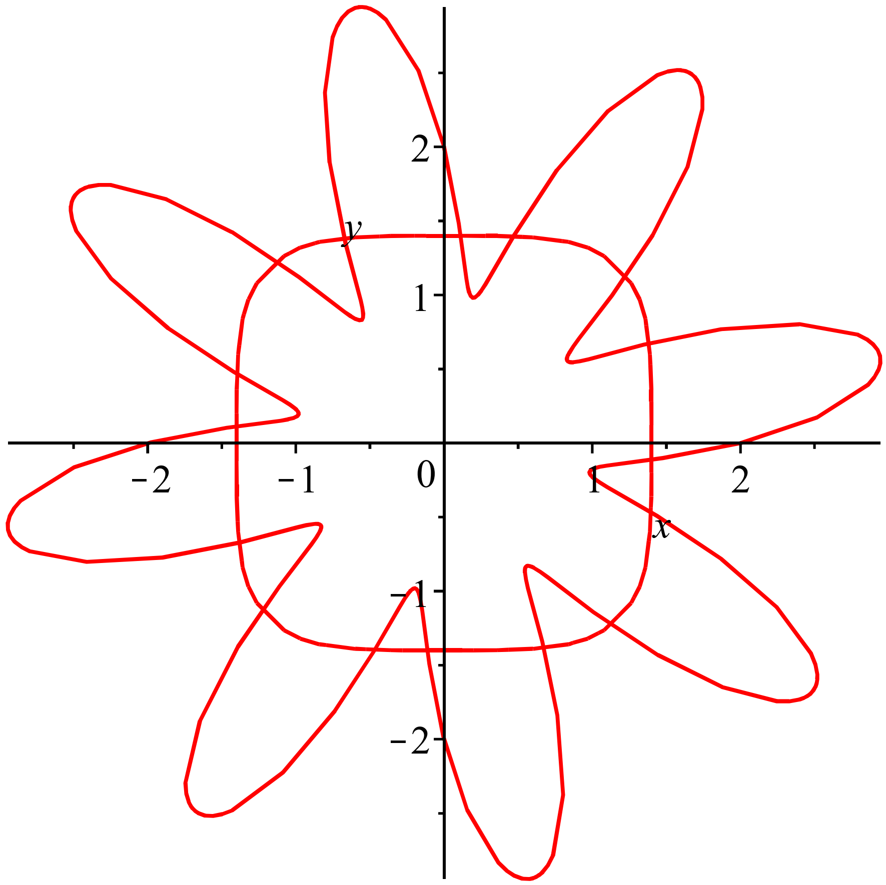
## Exercise 6

```
> with(plots):
```

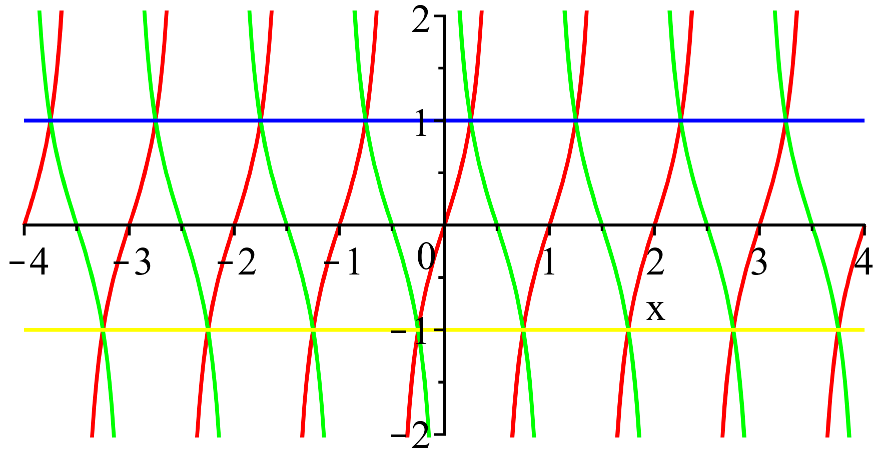
Warning, the name changecoords has been redefined

```
> display(
   implicitplot(x^4+y^4=4,x=-3..3,y=-3..3),
   plot([(2+sin(8*t))*cos(t),(2+sin(8*t))*sin(t),t=0..2*Pi])
  );
```




## Exercise 7

```
> plot(
  [tan(Pi*x),cot(Pi*x),-1,1],     # functions to plot
    x=-4..4,                      # horizontal range
      -2..2,                      # vertical range
    scaling=constrained,          # same scale on both axes
    discont=true                  # skip over discontinuities
  );
```




## Exercise 8

```
> restart;
> f := (t) -> ((2+sqrt(3))*t-1)/(2+sqrt(3)+t);
```

$$f := t \rightarrow \frac{\left(2+\sqrt{3}\right)t-1}{2+\sqrt{3}+t} \quad \textbf{(14)}$$

```
> g := (t) -> f(f(f(t)));
```

$$g := t \rightarrow f(f(f(t))) \quad \textbf{(15)}$$

```
> h := (t) -> g(g(t));
```

$$h := t \rightarrow g(g(t)) \quad \textbf{(16)}$$

```
> simplify(g(t));
```

$$\frac{t-1}{1+t} \quad \textbf{(17)}$$

```
> simplify(h(t));
```

$$-\frac{1}{t} \quad \textbf{(18)}$$

## Exercise 9

```
> y := sin(10*x);
```

$$y := \sin(10\,x) \tag{19}$$

```
> (diff(y,x$8)/y)^(1/8);
```

$$100000000^{1/8} \tag{20}$$

```
> simplify(%);
```

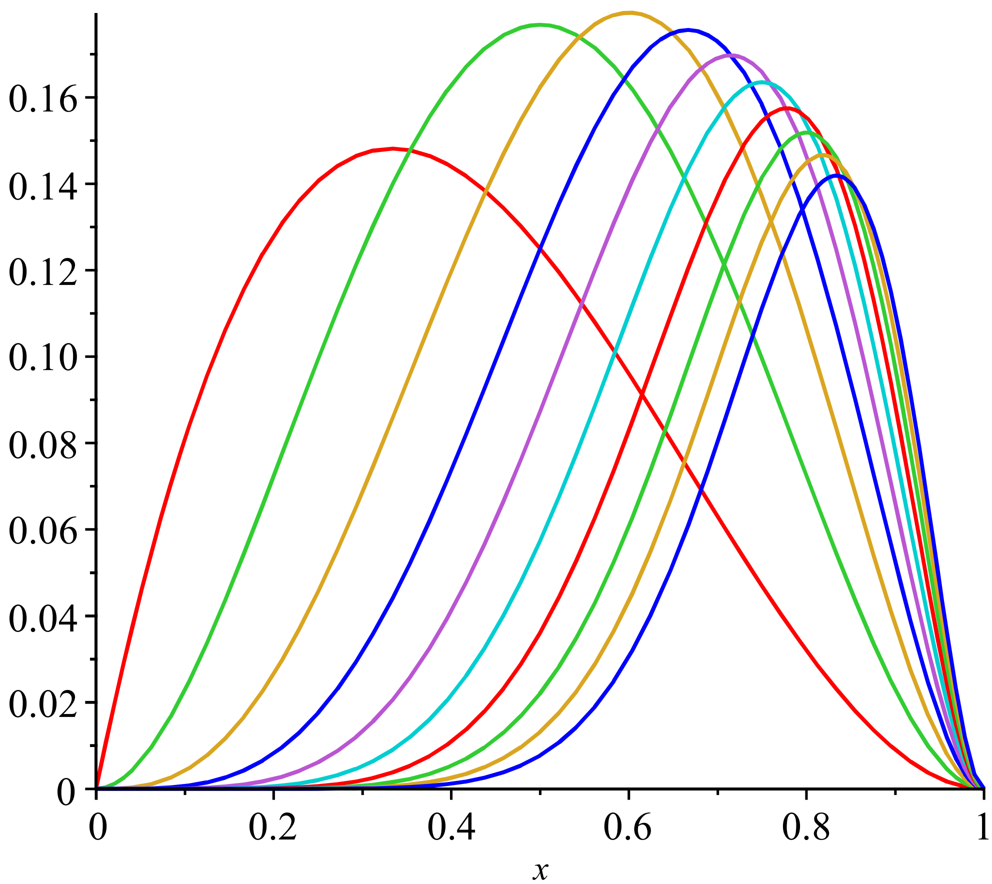
$$10 \tag{21}$$

---

**Exercise 10**

Here is the efficient method using the **seq()** command:

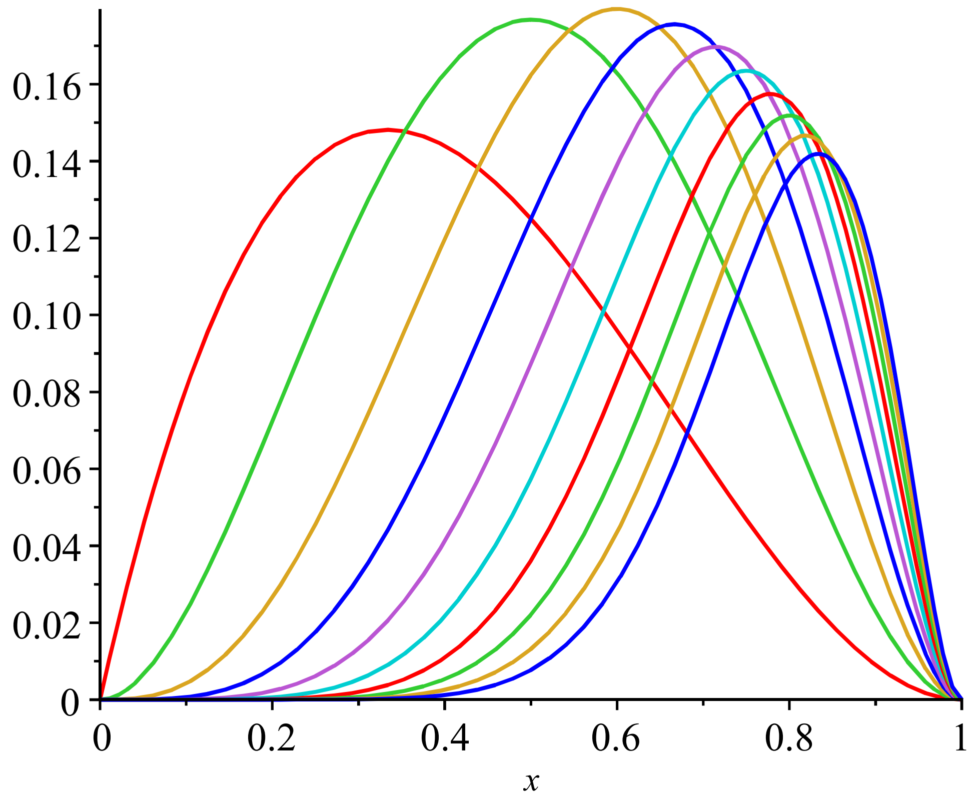
```
> plot([seq(n^(3/2) * x^n * (1-x)^2,n=1..10)],x=0..1);
```




Here is the less efficient method with more typing:

```
> plot([
  x * (1-x)^2,
  2^(3/2) * x^2 * (1-x)^2,
  3^(3/2) * x^3 * (1-x)^2,
  4^(3/2) * x^4 * (1-x)^2,
  5^(3/2) * x^5 * (1-x)^2,
```

```
6^(3/2) * x^6 * (1-x)^2,
7^(3/2) * x^7 * (1-x)^2,
8^(3/2) * x^8 * (1-x)^2,
9^(3/2) * x^9 * (1-x)^2,
10^(3/2) * x^10 * (1-x)^2
],x=0..1);
```

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**Exercise 11**

```
> y := x^4*(1-x)^4/(1+x^2);
```

$$y := \frac{x^4 (1-x)^4}{1+x^2} \quad (22)$$

```
> int(y,x);
```

$$\frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x - 4\arctan(x) \quad (23)$$

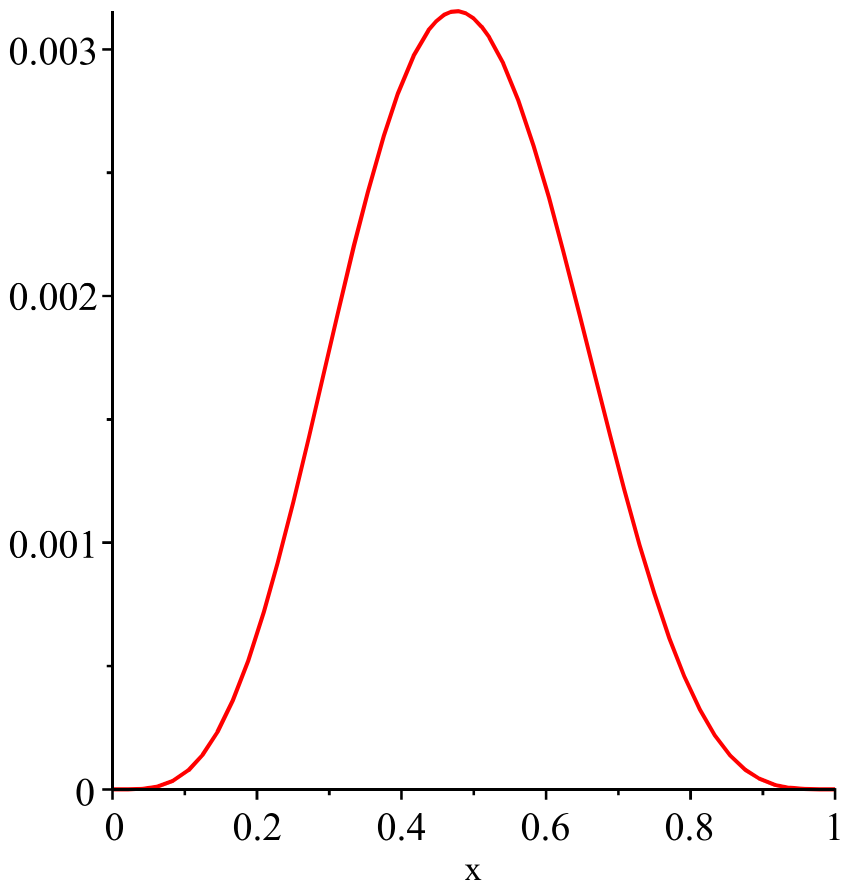
```
> int(y,x=0..1);
```

$$\frac{22}{7} - \pi \quad (24)$$

```
> simplify(diff(y,x));
```

$$\frac{2x^3(-1+x)^3(-2-x^2+4x+3x^3)}{(1+x^2)^2} \quad (25)$$

```
> plot(y,x=0..1);
```




The hump has position $p$ and height $h$, where $p$ is the root of $\frac{dy}{dx}=0$ near $x=0.5$ and $h$ is obtained by putting $x=p$ in $y$.

```
> p := fsolve(diff(y,x)=0,x=0.5);
```

$$p := 0.4758084110 \quad (26)$$

```
> h := subs(x=p, y);
```

$$h := 0.003155431532 \quad (27)$$

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## Exercise 12

```
> a := (n) -> evalf((1+1/n)^n/exp(1));
```

$$a := n \to evalf\left(\frac{\left(\frac{1}{n}+1\right)^n}{\mathrm{e}}\right) \quad (28)$$

```
> seq(a(n),n=1..100);
```

0.7357588824, 0.8277287427, 0.8720105272, 0.8981431670, 0.9154017711, 0.9276545006, 0.9368048855, 0.9438993732, 0.9495611401, 0.9541845268, 0.9580312773, 0.9612819623, 0.9640651903, 0.9664750464, 0.9685819515, 0.9704396614, 0.9720899237, 0.9735656522, 0.9748931474, 0.9760936771, 0.9771846268, **(29)**

0.9781803459, 0.9790927826, 0.9799319667, 0.9807063798, 0.9814232429,
0.9820887409, 0.9827082039, 0.9832862495, 0.9838268969, 0.9843336619,
0.9848096328, 0.9852575341, 0.9856797776, 0.9860785067, 0.9864556324,
0.9868128643, 0.9871517361, 0.9874736277, 0.9877797843, 0.9880713324,
0.9883492926, 0.9886145936, 0.9888680810, 0.9891105268, 0.9893426366,
0.9895650580, 0.9897783854, 0.9899831647, 0.9901799000, 0.9903690563,
0.9905510628, 0.9907263180, 0.9908951901, 0.9910580220, 0.9912151319,
0.9913668163, 0.9915133516, 0.9916549951, 0.9917919882, 0.9919245562,
0.9920529100, 0.9921772474, 0.9922977540, 0.9924146039, 0.9925279616,
0.9926379811, 0.9927448078, 0.9928485781, 0.9929494223, 0.9930474618,
0.9931428121, 0.9932355825, 0.9933258763, 0.9934137910, 0.9934994201,
0.9935828507, 0.9936641672, 0.9937434485, 0.9938207705, 0.9938962042,
0.9939698187, 0.9940416784, 0.9941118459, 0.9941803800, 0.9942473370,
0.9943127706, 0.9943767324, 0.9944392719, 0.9945004351, 0.9945602678,
0.9946188125, 0.9946761104, 0.9947322010, 0.9947871217, 0.9948409094,
0.9948935986, 0.9949452220, 0.9949958124, 0.9950454000

The first number greater than 0.99 occurs around the middle of the list, close to $n = 50$. We therefore look at a shorter list centred around that value:

```
> seq(a(n),n=48..52);
```

0.9897783854, 0.9899831647, 0.9901799000, 0.9903690563, 0.9905510628 **(30)**

We see that $a(50) = 0.9901799000$, and that this is the first number greater than 0.99.

Here is a slicker way:

```
> min(op(select(n -> (a(n)>0.99),[seq(n,n=1..100)])));
```

50 **(31)**