Maple Demonstration

Numerical calculation

Maple can be used as a numerical calculator:

```
> 2+2;
```

$$4 \quad (1.1)$$

```
> (3 + 7 + 10)*(1000 - 8)/(900 + 90 + 2) - 17;
```

$$3 \quad (1.2)$$

Unlike a normal calculator, it can work with numbers hundreds of digits long:

```
> 2^1000;
```

10715086071862673209484250490600018105614048117055336074437503883703510511\ (1.3)
24936122493198378815695858127594672917553146825187145285692314043598457\
75746985748039345677748242309854210746050623711418779541821530464749835\
81941267398767559165543946077062914571196477686542167660429831652624386\
837205668069376

```
> log[10](2^1000.);
```

$$301.0299957 \quad (1.4)$$

```
> 100!;
```

93326215443944152681699238856266700490715968264381621468592963895217599993\ (1.5)
22991560894146397615651828625369792082722375825118521091686400000000000\
0000000000000

Maple tries to do exact calculations where possible, so it will leave Pi as Pi rather than using a numerical approximation

```
> Pi;
```

$$\pi \quad (1.6)$$

You can ask for the approximate value using the function **evalf()**

```
> evalf(Pi);
```

$$3.141592654 \quad (1.7)$$

```
> evalf[1000](Pi);
```

3.1415926535897932384626433832795028841971693993751058209749445923078164062\ (1.8)
86208998628034825342117067982148086513282306647093844609550582231725359\
40812848111745028410270193852110555964462294895493038196442881097566593\
34461284756482337867831652712019091456485669234603486104543266482133936\

$$0726024914127372458700660631558817488152092096282925409171536436789259036001133053054882046652138414695194151160943305727036575959195309218611738193261179310511854807446237996274956735188575272489122793818301194912983367336244065664308602139494639522473719070217986094370277053921717629317675238467481846766940513200056812714526356082778577134275778960917363717872146844090122495343014654958537105079227968925892354201995611212902196086403441815981362977477130996051870721134999999837297804995105973173281609631859502445945534690830264252230825334468503526193118817101000313783875288658753320838142061717766914730359825349042875546873115956286388235378759375195778185778053217122680661300192787661119590921642019$$

Maple can calculate many functions that normal calculators cannot.

For example, here is the list of the first hundred prime numbers:

```
> seq(ithprime(i),i=1..100);
```

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199, 211, 223, 227, 229, 233, 239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 293, 307, 311, 313, 317, 331, 337, 347, 349, 353, 359, 367, 373, 379, 383, 389, 397, 401, 409, 419, 421, 431, 433, 439, 443, 449, 457, 461, 463, 467, 479, 487, 491, 499, 503, 509, 521, 523, 541 **(1.9)**

Symbolic calculation

Maple can work with symbolic expressions rather than numbers:

```
> (x-y)*(x^5+x^4*y+x^3*y^2+x^2*y^3+x*y^4+y^5);
```

$$(x-y)\,(x^5+x^4 y+x^3 y^2+x^2 y^3+x y^4+y^5) \tag{2.1}$$

```
> expand(%);
```

$$x^6-y^6 \tag{2.2}$$

```
> expand((x-y)*(x^5+x^4*y+x^3*y^2+x^2*y^3+x*y^4+y^5));
```

$$x^6-y^6 \tag{2.3}$$

```
> A := 2 *((u^2+1)^2 + (u^2-1)^2)/
       ((u^(-2)+1)^2 - (u^(-2)-1)^2);
```

$$A := \frac{2\left(\left(u^2+1\right)^2+\left(u^2-1\right)^2\right)}{\left(\frac{1}{u^2}+1\right)^2-\left(\frac{1}{u^2}-1\right)^2} \tag{2.4}$$

```
> simplify(A);
```

$$\left(u^4+1\right) u^2 \tag{2.5}$$

The Cauchy-Schwartz inequality

The Cauchy-Schwartz inequality says that for any real numbers u, v, w, x, y, z we have $(x\,u+y\,v+z\,w)^2 \le \left(x^2+y^2+z^2\right)\left(u^2+v^2+w^2\right)$.
In fact we have
$\left(x^2+y^2+z^2\right)\left(u^2+v^2+w^2\right) = (x\,u+y\,v+z\,w)^2+(x\,v-y\,u)^2+(y\,w-z\,v)^2+(z\,u-w\,x)^2$
$= (x\,u+y\,v+z\,w)^2 +$ some extra, positive terms.

To check this, we give names to the various terms:

```
> A := (x^2+y^2+z^2) * (u^2+v^2+w^2);
```

$$A := \left(x^2+y^2+z^2\right)\left(u^2+v^2+w^2\right) \tag{3.1}$$

```
> B := (x*u+y*v+z*w)^2;
```

$$B := (x\,u+y\,v+z\,w)^2 \tag{3.2}$$

```
> C := (x*v-y*u)^2 + (y*w-z*v)^2 + (z*u-x*w)^2;
```

$$C := (-y\,u+x\,v)^2+(-z\,v+y\,w)^2+(z\,u-w\,x)^2 \tag{3.3}$$

```
> expand(A);
```

$$u^2x^2+u^2y^2+u^2z^2+v^2x^2+v^2y^2+v^2z^2+w^2x^2+w^2y^2+w^2z^2 \tag{3.4}$$

```
> expand(B);
```

$$u^2x^2+2\,u\,v\,x\,y+2\,u\,w\,x\,z+v^2y^2+2\,v\,w\,y\,z+w^2z^2 \tag{3.5}$$

```
> expand(C);
```

$$u^2y^2+u^2z^2-2\,u\,v\,x\,y-2\,u\,w\,x\,z+v^2x^2+v^2z^2-2\,v\,w\,y\,z+w^2x^2+w^2y^2 \tag{3.6}$$

```
> expand(A-B-C);
```

$$0 \tag{3.7}$$

```
> unassign('A','B','C');
```

The cross-ratio

```
> chi := (a,b,c,d) -> (d-a)*(c-b)/((d-b)*(c-a));
```

$$\chi := (a, b, c, d) \mapsto \frac{(d-a)\,(c-b)}{(d-b)\,(c-a)} \tag{4.1}$$

```
> w := chi(a,b,c,d);
```

$$w := \frac{(d-a)\,(c-b)}{(d-b)\,(c-a)} \tag{4.2}$$

```
> x := chi(1/a,1/b,1/c,1/d);
```

$$x := \frac{\left(\frac{1}{d} - \frac{1}{a}\right)\left(\frac{1}{c} - \frac{1}{b}\right)}{\left(\frac{1}{d} - \frac{1}{b}\right)\left(\frac{1}{c} - \frac{1}{a}\right)} \tag{4.3}$$

```
> y := chi(c,d,a,b);
```

$$y := \frac{(b-c)\,(a-d)}{(b-d)\,(a-c)} \tag{4.4}$$

```
> z := chi(a+1,b+1,c+1,d+1);
```

$$z := \frac{(d-a)\,(c-b)}{(d-b)\,(c-a)} \tag{4.5}$$

```
> simplify(w-x); simplify(x-y); simplify(y-z);
```

$$0$$
$$0$$
$$0 \tag{4.6}$$

```
> unassign('chi','w','x','y','z');
```

Solving linear equations

```
> eqns := {x +   y +   z = 3,
           x + 2*y + 3*z = 6,
           x + 4*y + 9*z = 14};
```

$$eqns := \{x+y+z=3, x+2\,y+3\,z=6, x+4\,y+9\,z=14\} \tag{5.1}$$

```
> solve(eqns);
```

$$\{x=1, y=1, z=1\} \tag{5.2}$$

Solving nonlinear equations

```
> {sin(Pi*x) = 0, log(x)^3 = log(x)};
```

$$\{\ln(x)^3 = \ln(x), \sin(\pi x) = 0\} \tag{6.1}$$

```
> solve(%);
```

$$\{x=1\} \tag{6.2}$$

```
> log(log(x)) - log(x) + 1=0;
```

$$\ln(\ln(x)) - \ln(x) + 1 = 0 \tag{6.3}$$

```
> solve(%,{x});
```

$$\{x=\mathrm{e}\} \tag{6.4}$$

Equations with many solutions

Here is an example where there are many solutions, but Maple finds only one of them:

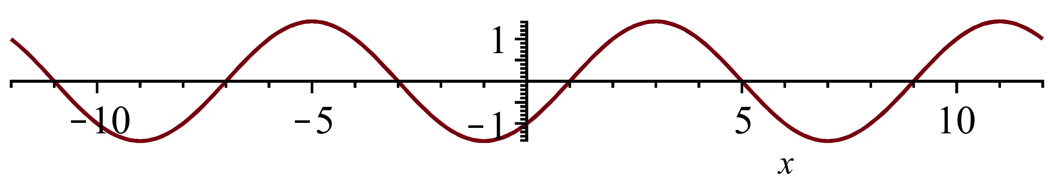
```
> sin(Pi*x/4)=cos(Pi*x/4);
```

$$\sin\left(\frac{\pi x}{4}\right) = \cos\left(\frac{\pi x}{4}\right) \quad (7.1)$$

```
> solve(%,{x});
```

$$\{x = 1\} \quad (7.2)$$

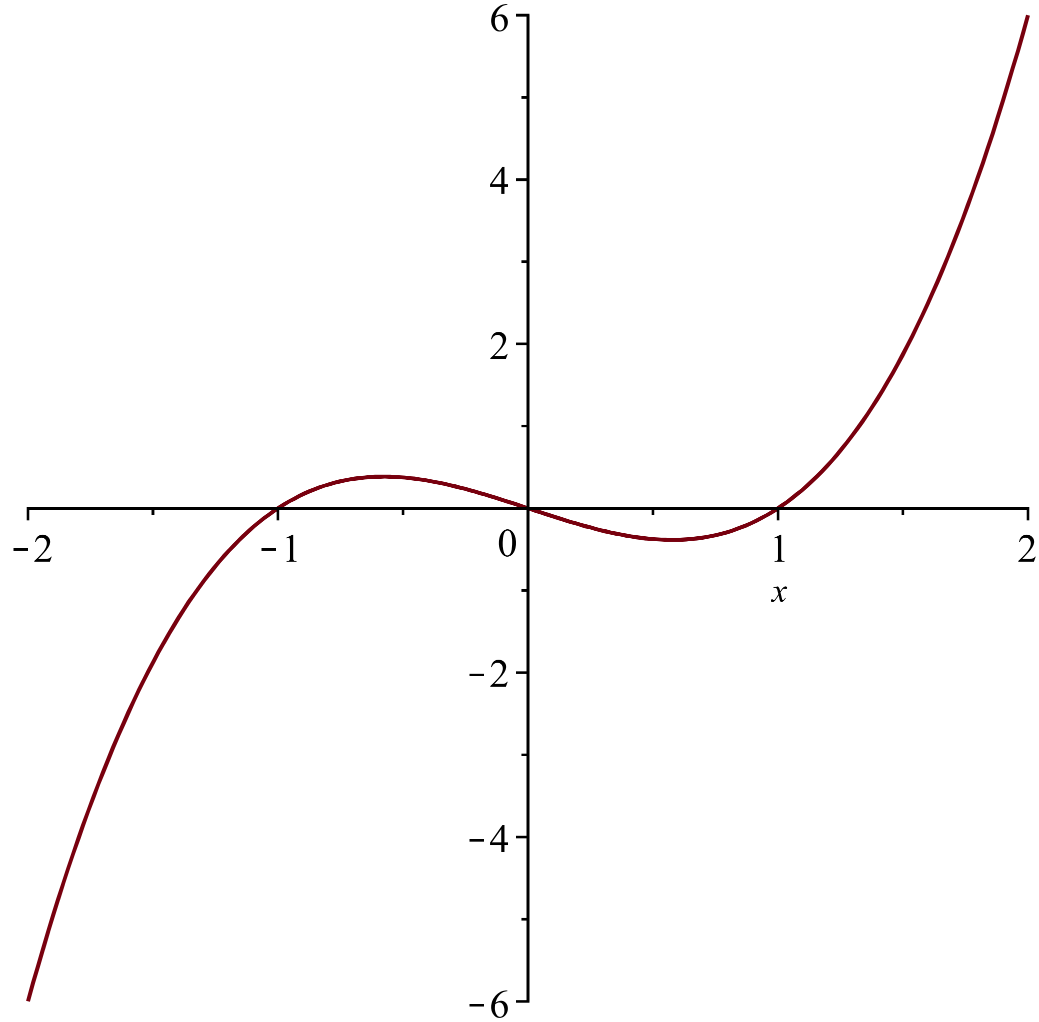
```
> plot(sin(Pi*x/4)-cos(Pi*x/4),x=-12..12,scaling=constrained);
```



Maple does not see the solutions $x = -7, -3, 5, 9, 13$ and so on. Later we will discuss how to fix this.

Plotting functions

```
> plot(x^3-x,x=-2..2);
```



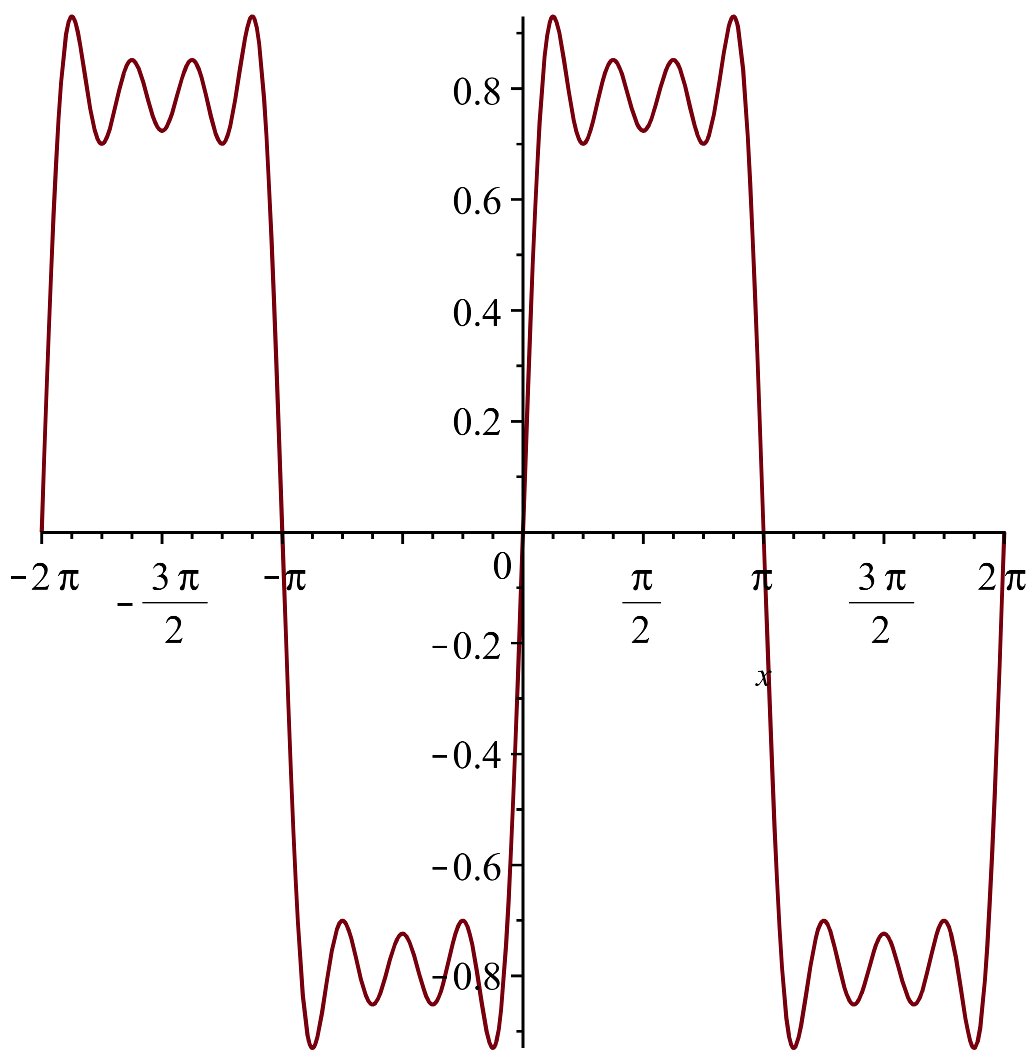

```
> f := x -> sin(x)+sin(3*x)/3+sin(5*x)/5+sin(7*x)/7;
```

$$f := x \mapsto \sin(x) + \frac{\sin(3\,x)}{3} + \frac{\sin(5\,x)}{5} + \frac{\sin(7\,x)}{7} \tag{8.1}$$

```
> f(x);
```

$$\sin(x) + \frac{\sin(3\,x)}{3} + \frac{\sin(5\,x)}{5} + \frac{\sin(7\,x)}{7} \tag{8.2}$$

```
> plot(f(x),x=-2*Pi..2*Pi);
```

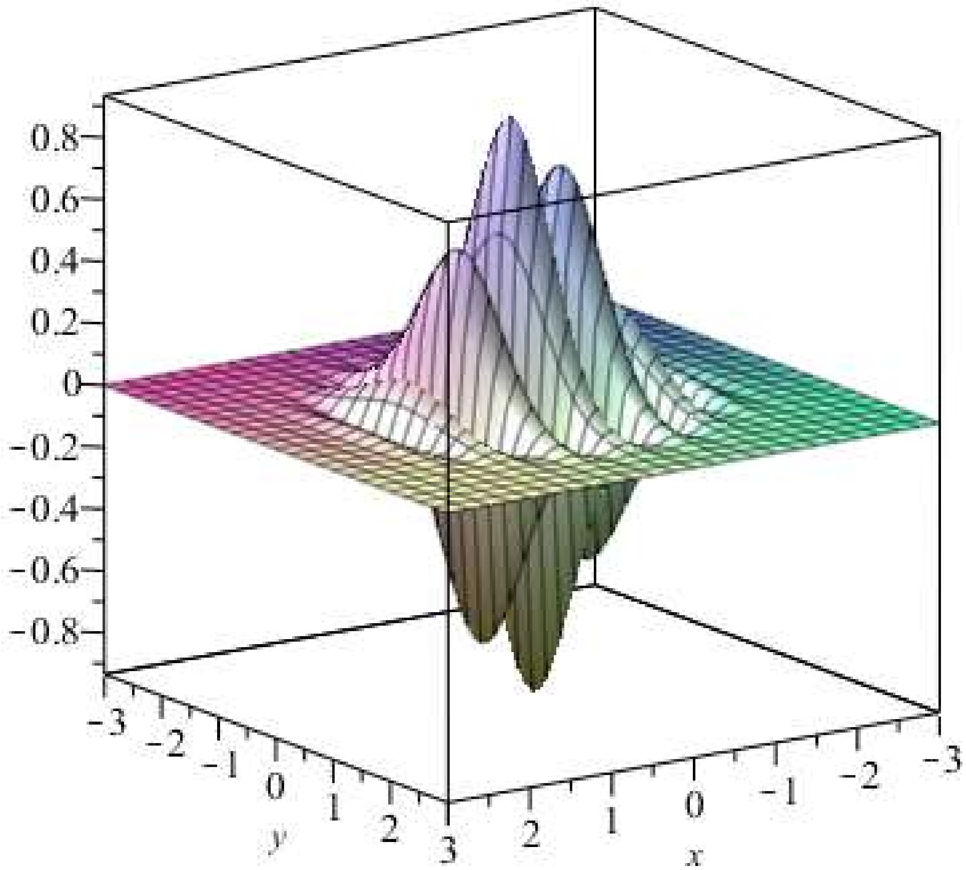


Three-dimensional plotting

```
> exp(-x^2-y^2)*sin(10*x);
```

$$e^{-x^2 - y^2} \sin(10\, x) \quad \textbf{(9.1)}$$

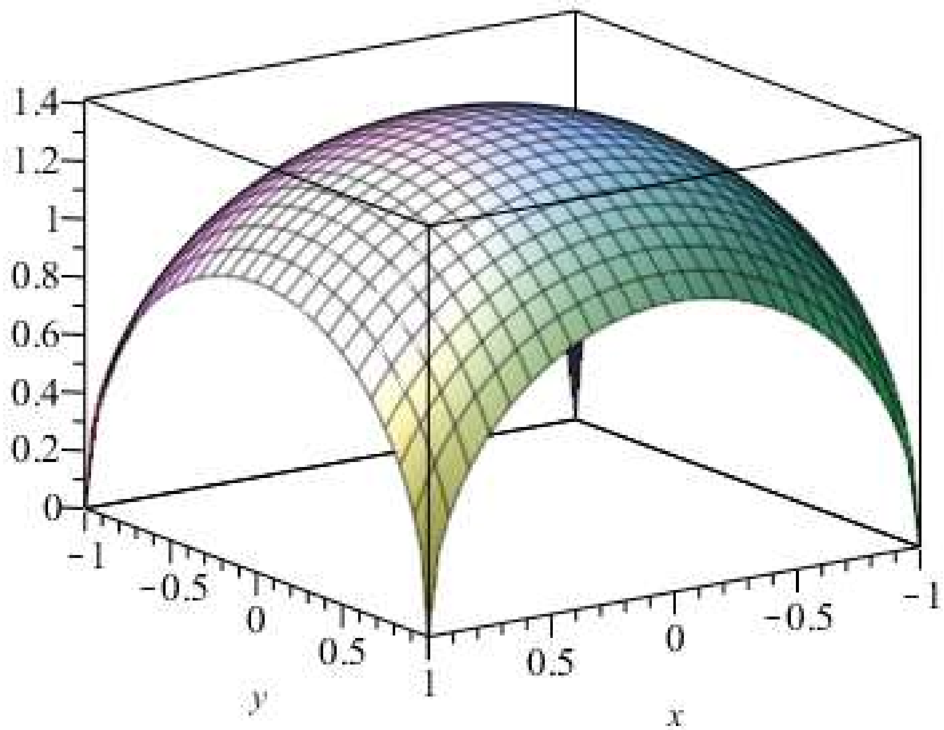
```
> plot3d(exp(-x^2-y^2)*sin(10*x),x=-3..3,y=-3..3);
```




```
> f := (x,y) -> sqrt(2 - x^2 - y^2);
```

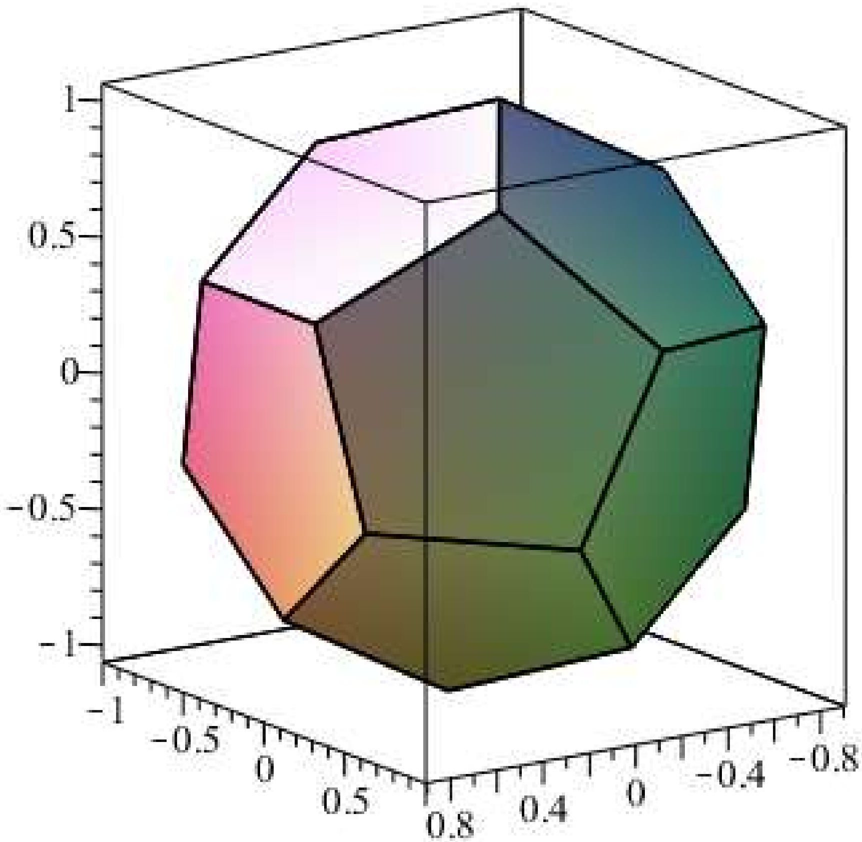
$$f := (x, y) \mapsto \sqrt{2 - x^2 - y^2} \tag{9.2}$$

```
> plot3d(f(x,y),x=-1..1,y=-1..1,
        axes=BOXED,scaling=constrained);
```



Other pictures

```
> with(plots):
> polyhedraplot(
   [0,0,0],
   polytype=dodecahedron,
   scaling=constrained
  );
```



Differentiation

```
> y := x^4+x^3+x^2+x+1;
```

$$y := x^4 + x^3 + x^2 + x + 1 \quad (11.1)$$

```
> Diff(y,x);
```

$$\frac{d}{dx}\left(x^4 + x^3 + x^2 + x + 1\right) \quad (11.2)$$

```
> diff(y,x);
```

$$4\,x^3 + 3\,x^2 + 2\,x + 1 \quad (11.3)$$

```
> Diff(y,x,x,x);
```

$$\frac{d^3}{dx^3}\left(x^4 + x^3 + x^2 + x + 1\right) \quad (11.4)$$

```
> diff(y,x,x,x);
```

$$24\,x + 6 \quad (11.5)$$

```
> unassign('y');
```

```
> f := (x) -> (2*x^2+3)/(x^2+2);
```

$$f := x \mapsto \frac{2\,x^2+3}{x^2+2} \tag{11.6}$$

```
> diff(f(x),x);
```

$$\frac{4\,x}{x^2+2} - \frac{2\,(2\,x^2+3)\,x}{(x^2+2)^2} \tag{11.7}$$

```
> simplify(diff(f(x),x));
```

$$\frac{2\,x}{(x^2+2)^2} \tag{11.8}$$

```
> f := (x) -> sqrt(2*Pi)*x^(x+1/2)*exp(-x);
```

$$f := x \mapsto \sqrt{2\,\pi}\;x^{x+\frac{1}{2}}\,e^{-x} \tag{11.9}$$

```
> simplify(diff(f(x),x));
```

$$\frac{\sqrt{2}\,\sqrt{\pi}\;x^{x-\frac{1}{2}}\,e^{-x}\,(2\ln(x)\,x+1)}{2} \tag{11.10}$$

```
> evalf(50!);
```

$$3.041409320\;10^{64} \tag{11.11}$$

```
> evalf(f(50));
```

$$3.036344593\;10^{64} \tag{11.12}$$

```
> seq(evalf(n!/f(n)),n=2..22);
```

1.042207121, 1.028064518, 1.021008303, 1.016783985, 1.013972848, 1.011967757, 1.010465651, 1.009298426, 1.008365359, 1.007602428, 1.006966997, 1.006429575, 1.005969115, 1.005570189, 1.005221239, 1.004913427, 1.004639885, 1.004395190, 1.004175011, 1.003975836, 1.003794800 **(11.13)**

Integration

```
> Int(x^4*exp(-x),x);
```

$$\int x^4\,e^{-x}\,dx \tag{12.1}$$

```
> simplify(int(x^4*exp(-x),x));
```

$$-\left(x^4+4x^3+12x^2+24x+24\right)e^{-x} \tag{12.2}$$

```
> Int(x/(x^4-1),x);
```

$$\int \frac{x}{x^4-1}\,dx \tag{12.3}$$

```
> int(x/(x^4-1),x);
```

$$\frac{\ln(x-1)}{4}+\frac{\ln(x+1)}{4}-\frac{\ln\left(x^2+1\right)}{4} \tag{12.4}$$

```
> Int(log(x)^6,x);
```

$$\int \ln(x)^6\,dx \tag{12.5}$$

```
> simplify(int(log(x)^6,x));
```

$$x\left(\ln(x)^6-6\ln(x)^5+30\ln(x)^4-120\ln(x)^3+360\ln(x)^2-720\ln(x)+720\right) \tag{12.6}$$

```
> Int(exp(-x^2),x=-infinity..infinity);
```

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx \tag{12.7}$$

```
> value(%);
```

$$\sqrt{\pi} \tag{12.8}$$