Formulae for PMA101

No formula sheets etc will be permitted in the exam. You should remember (or be able to derive) all the formulae listed below. You should also remember all the rules and methods in the notes, which are not explicitly listed here.

$$
\begin{aligned}
\exp(x+y) &= \exp(x)\exp(y) & \log(xy) &= \log(x)+\log(y) \\
\exp(x-y) &= \exp(x)/\exp(y) & \log(x/y) &= \log(x)-\log(y) \\
\exp(0) &= 1 & \log(1) &= 0 \\
\exp(-x) &= 1/\exp(x) & \log(1/y) &= -\log(y) \\
\exp(nx) &= \exp(x)^n & \log(y^n) &= n\log(y) \\
\exp(x) &= e^x
\end{aligned}
$$

$$\log_a(y) = \log(y)/\log(a) = \text{ the number } x \text{ such that } a^x = y\ .$$

$$
\begin{aligned}
\sinh(x) &= (e^x - e^{-x})/2 & \operatorname{csch}(x) &= 1/\sinh(x) = 2/(e^x - e^{-x}) \\
\cosh(x) &= (e^x + e^{-x})/2 & \operatorname{sech}(x) &= 1/\cosh(x) = 2/(e^x + e^{-x}) \\
\tanh(x) &= \sinh(x)/\cosh(x) = (e^x - e^{-x})/(e^x + e^{-x}) & \coth(x) &= 1/\tanh(x) = (e^x + e^{-x})/(e^x - e^{-x}).
\end{aligned}
$$

$$
\begin{aligned}
\tan(\theta) &= \sin(\theta)/\cos(\theta) & \cot(\theta) &= \cos(\theta)/\sin(\theta) \\
\sec(\theta) &= 1/\cos(\theta) & \csc(\theta) &= 1/\sin(\theta).
\end{aligned}
$$

$$
\begin{aligned}
\sin(x+y) &= \sin(x)\cos(y) + \cos(x)\sin(y) & \sin(x-y) &= \sin(x)\cos(y) - \cos(x)\sin(y) \\
\cos(x+y) &= \cos(x)\cos(y) - \sin(x)\sin(y) & \cos(x-y) &= \cos(x)\cos(y) + \sin(x)\sin(y) \\
\sin(2x) &= 2\sin(x)\cos(x) & \cos(2x) &= \cos(x)^2 - \sin(x)^2 \\
& & &= 2\cos(x)^2 - 1 = 1 - 2\sin(x)^2 \\
\sin(x)^2 &= \tfrac{1}{2} - \tfrac{1}{2}\cos(2x) & \cos(x)^2 &= \tfrac{1}{2} + \tfrac{1}{2}\cos(2x)
\end{aligned}
$$

θ	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$
$\pi/2$	1	0	∞
$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$

$$
\begin{aligned}
\exp'(x) &= \exp(x) & \log'(x) &= 1/x \\
\sinh'(x) &= \cosh(x) & \operatorname{arcsinh}'(x) &= (1+x^2)^{-1/2} \\
\cosh'(x) &= \sinh(x) & \operatorname{arccosh}'(x) &= (x^2-1)^{-1/2} \\
\tanh'(x) &= \operatorname{sech}(x)^2 = 1 - \tanh(x)^2 & \operatorname{arctanh}'(x) &= (1-x^2)^{-1} \\
\sin'(x) &= \cos(x) & \arcsin'(x) &= (1-x^2)^{-1/2} \\
\cos'(x) &= -\sin(x) & \arccos'(x) &= -(1-x^2)^{-1/2} \\
\tan'(x) &= \sec(x)^2 = 1 + \tan(x)^2 & \arctan'(x) &= (1+x^2)^{-1}
\end{aligned}
$$

$$\int \exp(x)\,dx = \exp(x) \qquad \int \log(x)\,dx = x\log(x) - x$$

$$\int \sin(x)\,dx = -\cos(x) \qquad \int \cos(x)\,dx = \sin(x)$$

$$\int \sin(x)^2\,dx = \frac{2x - \sin(2x)}{4} \qquad \int \cos(x)^2\,dx = \frac{2x + \sin(2x)}{4}$$

$$\int \frac{dx}{1+x^2} = \arctan(x) \qquad \int \frac{dx}{1-x^2} = \operatorname{arctanh}(x)$$

$$\int \frac{dx}{\sqrt{1+x^2}} = \operatorname{arcsinh}(x) \qquad \int \frac{dx}{\sqrt{1-x^2}} = \arcsin(x)$$

$$\int \frac{dx}{\sqrt{x^2-1}} = \operatorname{arccosh}(x) \qquad \int \tan(x)\,dx = -\log(\cos(x))$$

$$\int a^x\,dx = a^x / \log(a)$$

$$\int x^k\,dx = x^{k+1}/(k+1)$$

$$\int (x-a)^{-1}\,dx = \log(x-a)$$

$$\int (x-a)^{-k}\,dx = (x-a)^{1-k}/(1-k) \qquad \text{(for } k > 1\text{)}$$

You *do not* need to remember the following formulae:

$$\int \frac{dx}{ax^2+bx+c} = \begin{cases} \frac{2}{\sqrt{4ac-b^2}} \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right) & \text{if } 4ac > b^2 \\ \frac{-2}{\sqrt{b^2-4ac}} \operatorname{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right) & \text{if } 4ac < b^2 \end{cases}$$

$$\int \frac{dx}{\sqrt{ax^2+bx+c}} = \log(2\sqrt{a^2x^2+abx+ac} + 2ax + b)/\sqrt{a}$$